

ASME B89.7.3.1-2001

# GUIDELINES FOR DECISION RULES: CONSIDERING MEASUREMENT UNCERTAINTY IN DETERMINING CONFORMANCE TO SPECIFICATIONS

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A N A M E R I C A N N A T I O N A L S T A N D A R D

# **GUIDELINES FOR DECISION RULES: CONSIDERING MEASUREMENT UNCERTAINTY IN DETERMINING CONFORMANCE TO SPECIFICATIONS**

## **ASME B89.7.3.1-2001**

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## FOREWORD

The intent of these guidelines is to facilitate the development of understanding between suppliers and customers regarding measurement uncertainty in the decision to accept or to reject a product. Metrologists are continuously faced with the task of making decisions in the presence of measurement uncertainty. To formalize this task, procedures known as decision rules have been developed. A decision rule is a prescription for the acceptance or rejection of products based on the measurement result of a characteristic of the product, the permissible variation associated with that characteristic, and the uncertainty of the measurement result. For workpieces, the permissible variation is commonly called the tolerance; for instruments it is often given by the specification limits or maximum permissible error (MPE). The terminology of ISO 14253-1 has been adopted and the permitted variation of a product's characteristic is referred to as the specification zone. This document is intended to provide guidance on decision rules and their implementation.

A related document, ASME B89.7.2-1999, Dimensional Measurement Planning, specifies requirements for preparation and approval of dimensional measurement plans and for the use of approved plans in making dimensional measurements. The dimensional measurement plan must contain or reference all information for making measurements, including specification of a decision rule. ASME B89.7.3.1 serves as a resource to the dimensional measurement planner by providing terminology and specifying the requirements for decision rules for use in dimensional measurement plans.

The Guide to the Expression of Uncertainty in Measurement, (GUM), NCSL Z540-2-1997 provides a unified means of evaluating and expressing the uncertainty of a measurement result; consequently the calculational details of evaluating the uncertainty of a measurement result will not be discussed. Unless otherwise stated, the term "measurement uncertainty" will be used to mean the expanded uncertainty,  $U$ , with a coverage factor of two, which is the most common coverage factor used nationally and internationally.

Although all traceable measurement results include an uncertainty statement not all measurement results involve decision rules. (See ISO International Vocabulary of Basic and General Terms in Metrology.) Many calibrations, particularly at National Measurement Institutes (NMIs), typically state a description of the measurement, its result, and its uncertainty; decision rules are not involved since there are no specifications. Most products, however, have stated specifications and a decision must be reached regarding the product's characteristic relative to its stated specifications.

The decision rule in use should be well documented to prevent ambiguity in the acceptance or rejection of product. The selection of a particular decision rule is ultimately a business decision; some of the factors to be considered are outlined in nonmandatory Appendices A and D.

The concept of a decision rule has a long history and over the years has developed many variations including "gauge maker's rule," "test accuracy ratio (TAR)," "test uncertainty ratio (TUR)," "four-to-one rule," "gauging ratio," "guard bands," "gauging limit," and many more. Most of these terms were defined before the development of the GUM and hence concepts such as "accuracy" or "uncertainty" were nebulously defined. One of the motivations of these guidelines is to explicitly define the decision rule concept and have some well-documented decision rules that can be referenced. Consequently, these guidelines have encapsulated some of the commonly used procedures and their specifically-named decision rules.

The terminology used in these guidelines is consistent with national and international standards whenever possible. Descriptors such as "stringent" and "relaxed," used in describing conformance and nonconformance, have been carefully chosen. For example, stringent acceptance is meant to imply both a *decrease* in the acceptance zone width and an *increase* in confidence that a measurement result in this zone is associated with an in-specification product. Similarly, stringent rejection results in a decreased size of the rejection zone while increasing the confidence that a measurement result in this zone is associated with an out-of-specification product. The converse situation applies to relaxed acceptance and rejection.

The decision rules formulated using these guidelines ensure a self-consistent procedure for an organization to accept or to reject products. The situation becomes more complicated when two or more parties are involved, commonly a supplier and a customer, each of which is using a different measurement system with a different uncertainty and possibly using a different decision rule (this topic is very briefly discussed in nonmandatory Appendix A). Such a situation has the potential for conflicting decisions by the different parties, and conflict resolution is outside the scope of this document. When using decision rules in multi-party commerce, it is prudent to anticipate the potential conflicts that can arise (which depend on the details of the decision rules and the measurement systems involved) and agree upon a conflict resolution procedure prior to performing measurements.

Comments and suggestions for improvement of this Standard are welcomed. They should be addressed to: ASME, Three Park Avenue, New York, NY 10016-5990

This Standard was approved by the American National Standards Institute on December 11, 2001.

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**S. Phillips**, NIST, Gaithersburg, Maryland  
**J. Raja**, UNCC, Charlotte, North Carolina  
**P. Stein**, P. G. Stein Consultants, Pennington, New Jersey

### PERSONNEL OF WORKING GROUP B89.7.3: DECISION RULES

**S. Phillips**, *Chair*, NIST, Gaithersburg, Maryland  
**J. Buttress**, Hutchinson Technology, Hutchinson, Minnesota  
**T. Carpenter**, US Air Force Metrology Labs, Newark, Ohio  
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Edition:	Cite the applicable edition of the standard for which the interpretation is being requested.
Question:	Phrase the question as a request for an interpretation of a specific requirement suitable for general understanding and use, not as a request for an approval of a proprietary design or situation.

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## **ABSTRACT**

These guidelines provide suggestions for decision rules when considering measurement uncertainty in determining conformance to specifications. Applying these guidelines can assist businesses in avoiding disagreements with customers and suppliers about conformance to specifications and in managing costs associated with conformance decisions.

# GUIDELINES FOR DECISION RULES: CONSIDERING MEASUREMENT UNCERTAINTY IN DETERMINING CONFORMANCE TO SPECIFICATIONS

## 1 SCOPE

These guidelines provide terminology and specify the content that must be addressed when stating a decision rule used for deciding the acceptance or rejection of a product according to specification.

## 2 DEFINITIONS

*decision rule:* a documented rule, meeting the requirements of section 3 of these guidelines, that describes how measurement uncertainty will be allocated with regard to accepting or rejecting a product according to its specification and the result of a measurement.

*binary decision rule:* a decision rule with only two possible outcomes, either acceptance or rejection.<sup>1</sup>

*specification zone (of an instrument or workpiece):* the set of values of a characteristic between, and including, the specification limits.<sup>2, 3, 4</sup>

*measurand:* particular quantity subject to measurement. See VIM, 2.6.<sup>5</sup>

*expanded uncertainty:* quantity defining an interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand. See GUM, 2.3.5.

*uncertainty interval (of a measurement):* the set of values of a characteristic about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand.<sup>6, 7</sup>

*N:1 decision rule:* a situation where the width of the specification zone is at least  $N$  times larger than the uncertainty interval for the measurement result.<sup>8</sup>

*acceptance zone:* the set of values of a characteristic, for a specified measurement process and decision rule, that results in product acceptance when a measurement result is within this zone.<sup>9</sup>

*rejection zone:* the set of values of a characteristic, for a specified measurement process and decision rule, that results in product rejection when a measurement result is within this zone.<sup>10</sup>

*transition zone:* the set of values of a characteristic, for a specified measurement process and decision rule, that is neither in the acceptance zone nor rejection zone.<sup>11</sup>

<sup>1</sup> A binary decision rule does not have any transition zones (see 2.10).

<sup>2</sup> The width of the specification zone is a positive number.

<sup>3</sup> In the case of workpieces, the width of the specification zone is identical to the tolerance.

<sup>4</sup> Specification zone is equivalent to "tolerance interval" or "tolerance zone" defined in ISO 3534-2.

<sup>5</sup> The specification of a measurand may require statements about such quantities as time, temperature, and pressure.

<sup>6</sup> The width of the uncertainty interval is typically twice the expanded uncertainty.

<sup>7</sup> The uncertainty interval for the mean of repeated measurements may decrease with increasing numbers of measurements.

<sup>8</sup> A common example is the 4:1 ratio.

<sup>9</sup> When claiming product acceptance, it is important to state the decision rule; e.g., "acceptance using the XX rule."

<sup>10</sup> When claiming product rejection, it is important to state the decision rule; e.g., "rejection using the XX rule."

<sup>11</sup> There may be more than one transition zone; each should be separately labeled.

**guard band:** the magnitude of the offset from the specification limit to the acceptance or rejection zone boundary.<sup>12, 13, 14, 15, 16, 17</sup>

**simple acceptance:** the situation when the acceptance zone equals and is identical to the specification zone.

**simple rejection:** the situation when the rejection zone consists of all values of the characteristic outside the specification zone.

**stringent acceptance:** the situation when the acceptance zone is reduced from the specification zone by a guard band(s). See Fig. 1.<sup>18, 19</sup>

**relaxed rejection:** the situation when the rejection zone is partially inside the specification zone by the amount of a guard band. See Fig. 1.<sup>18</sup>

**relaxed acceptance:** the situation when the acceptance zone is increased beyond the specification zone by a guard band.<sup>20</sup>

<sup>12</sup> The symbol  $g$  is deliberately used for the guard band, instead of the symbol  $U$  employed in ISO 14253-1 since  $U$  is reserved for the expanded uncertainty which is associated with a measurement result and hence it is confusing to attach  $U$  to a specification limit. The evaluation of  $U$  is a technical issue, while the evaluation of  $g$  is a business decision.

<sup>13</sup> The guard band is usually expressed as a percentage of the expanded uncertainty, i.e., a 100% guard band has the magnitude of the expanded uncertainty  $U$ .

<sup>14</sup> Two-sided guard banding occurs when a guard band is applied to both the upper and lower specification limits. (In some exceptional situations the guard band applied within the specification zone,  $g_{in}$ , could be different at the upper specification limit and at the lower specification limit. This would reflect a different risk assessment associated with an upper or lower out-of-specification condition depending on whether the characteristic was larger or smaller than allowed by the specification zone.) If both the upper and lower guard bands are the same size then this is called symmetric two-sided guard banding.

<sup>15</sup> A guard band is sometimes distinguished as the upper or lower guard band, associated with the upper or lower specification limit. Subscripts are sometimes attached to the guard band notation,  $g$ , to provide clarity, e.g.,  $g_{up}$  and  $g_{lo}$ . See Fig. 1.

<sup>16</sup> The guard band,  $g$ , is always a positive quantity; its location, e.g., inside or outside the specification zone, is determined by the type of acceptance or rejection desired. See Section 4.

<sup>17</sup> While these guidelines emphasize the use of guard bands, an equivalent methodology is to use gauging limits as in ASME B89.7.2-1999.

<sup>18</sup> Stringent acceptance and relaxed rejection occur together in a binary decision rule.

<sup>19</sup> The stringent acceptance zone is analogous to the conformance zone described in ISO 14253-1.

<sup>20</sup> Relaxed acceptance and stringent rejection occur together in a binary decision rule.

**stringent rejection:** the situation when the rejection zone is increased beyond the specification zone by a guard band.<sup>20</sup>

**mean measurement result:** results of repeated measurements are arithmetically averaged to yield a mean measurement result. The mean result is used to determine acceptance or rejection.

**data rejection with cause:** repeated measurements may indicate that one or more measurement results significantly deviate from the rest of the results of measurement. If the measurement procedure has a documented policy for addressing measurement rejection then this policy takes precedence. Otherwise, measurement results may only be rejected if a physical cause can be established. Examples of physical causes for measurement rejection include: improper instrument settings, loose or improperly fixtured components, known transient events such as vibrations caused by doors slamming.

### 3 REQUIREMENTS FOR DECISION RULES

#### 3.1 Zone Identification

A decision rule must have a well-documented method of determining the location of the acceptance, rejection, and any transition zones.

#### 3.2 Decision Outcome

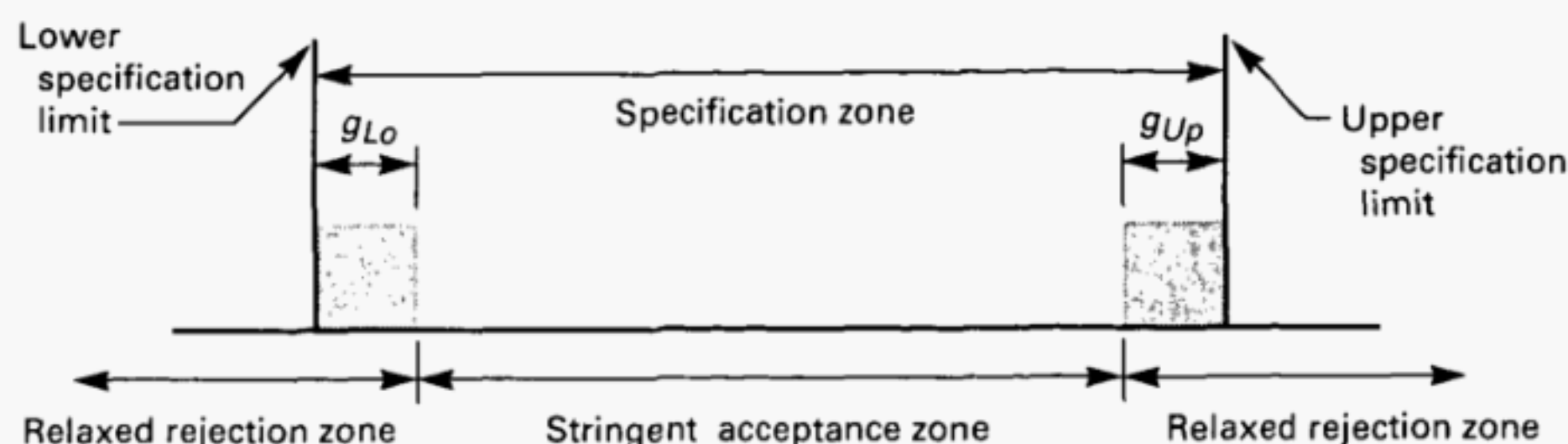
Each zone of a decision rule must correspond to a documented decision that will be implemented should the result of measurement lie in that zone. While this is automatic for the acceptance and rejection zones by definition, any transition zones must have their corresponding decision outcome documented.

#### 3.3 Repeated Measurements

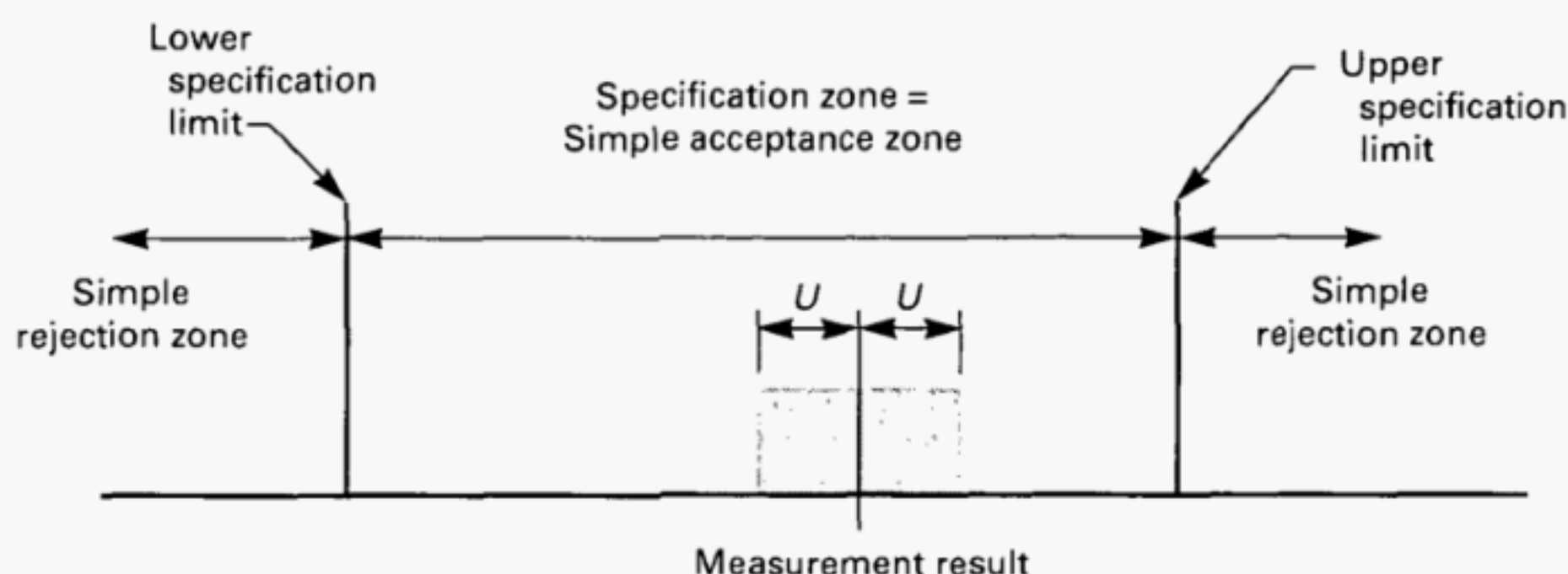
A decision rule must state the procedure for addressing repeated measurements of the same characteristic on the same workpiece or instrument. See Appendix B for further discussion of this issue.

#### 3.4 Data Rejection

A decision rule must state the procedure for allowing data rejection with cause, that is, rejection of "outliers." See Appendix C for further discussion of outliers.



**FIG. 1 AN EXAMPLE OF GUARD BANDS USED FOR CREATING A BINARY DECISION RULE WITH STRINGENT ACCEPTANCE AND RELAXED REJECTION ZONES**



GENERAL NOTE: The measurement uncertainty interval is of width  $2U$ , where  $U$  is the expanded uncertainty, and the uncertainty interval is no larger than one-fourth the product's specification zone. The measurement result shown verifies product acceptance.

**FIG. 2 AN EXAMPLE OF SIMPLE ACCEPTANCE AND REJECTION USING A 4:1 RATIO**

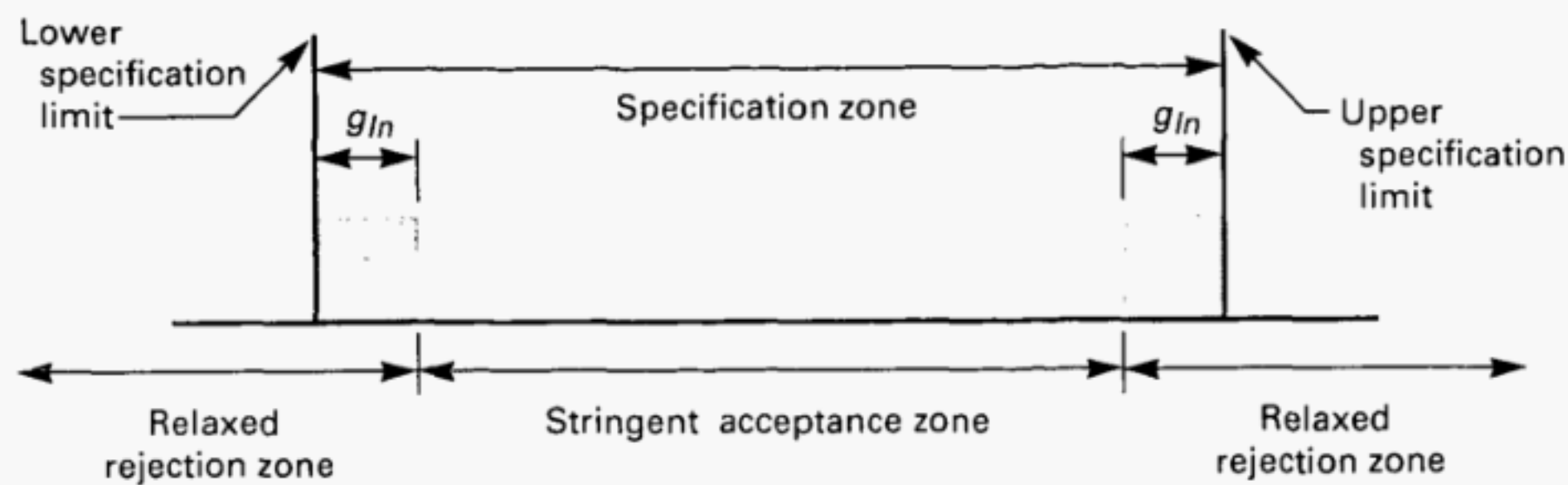
## 4 ACCEPTANCE AND REJECTION ZONES IN DECISION RULES

### 4.1 Simple Acceptance and Rejection Using an N:1 Decision Rule

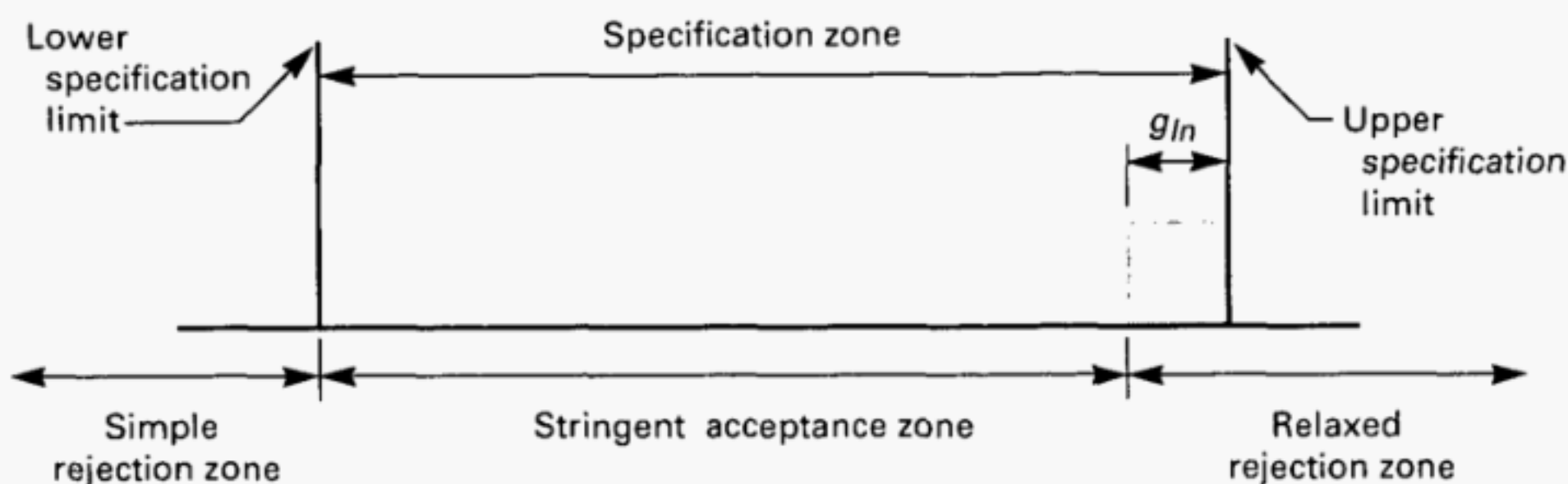
This is the most common form of acceptance and rejection used in industry and is the descendant of MIL-STD 45662A. Simple acceptance means that product conformance is verified<sup>21</sup> if the measurement result lies in the specification zone and rejection is verified otherwise (see Fig. 2), provided that the magnitude of the measurement uncertainty interval is no larger than the fraction  $1/N$  of the specification zone. In recent years, as tolerances have become increasingly tighter, the well-known ten-to-one ratio has transitioned to a more commonly used ratio of four-to-one (see MIL-STD 45662A) or even three-to-one (see International Standard 10012-1). A 4:1 decision rule means the uncertainty interval associated with the measurement

result should be no larger than one-fourth of the allowable product variation, which requires the expanded uncertainty,  $U$ , to be no larger than one-eighth of the specification zone. Once the uncertainty requirement is satisfied, then the product is accepted if the measurement result lies within the specification zone and rejected otherwise. Note that instrumentation is sometimes specified by a maximum permissible error (MPE), which places a limit on the magnitude of the error regardless of sign. Hence the specification zone has a width of twice the MPE, i.e.,  $\pm MPE$ , and a four-to-one ratio requires the expanded uncertainty to be one-fourth the MPE value; see Appendix D for further details. While the simple acceptance and rejection approach is straightforward, difficulties develop for measurement results close to the specification limits. Even using the mean of repeated measurements, if the mean result is near the specification limit there may be a significant chance that a product characteristic with simple acceptance verified is actually out-of-specification and vice versa. To address this issue, an alternative decision rule based on "guard banding" can increase confidence in acceptance decisions.

<sup>21</sup> The term "verified" or "verification" is used in the ISO Guide 25 sense; specifically avoided is the term "proven to conformance" as only a statistical confidence level is asserted, not a proof in the mathematical sense of the word.



(a) Symmetric Two-Sided Guard Banding



(b) One-Sided Guard Banding

GENERAL NOTE: Products are accepted if the measurement result is within the acceptance zone.

**FIG. 3 STRINGENT ACCEPTANCE AND RELAXED REJECTION EXAMPLES**

#### 4.2 Stringent Acceptance and Relaxed Rejection Using a Z% Guard Band

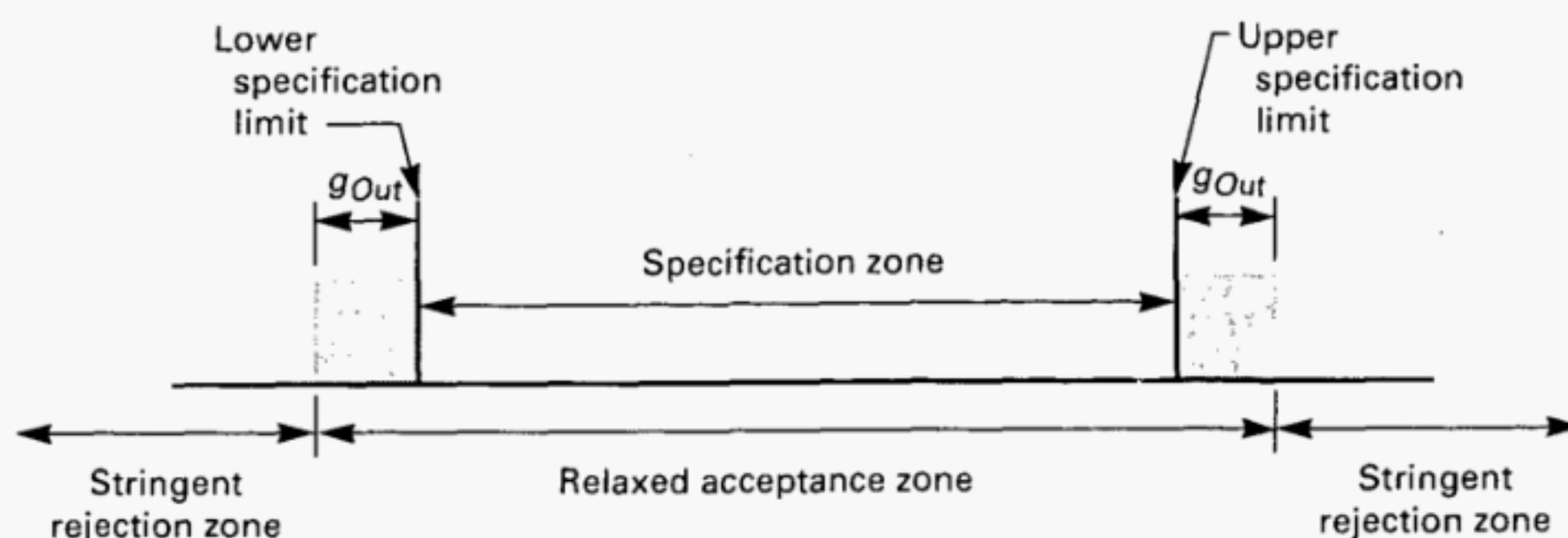
Stringent acceptance increases confidence in product quality by reducing the probability of accepting an out-of-specification product through the use of guard banding. The acceptance zone is created by reducing the specification zone by the guard band amount(s) as deemed necessary for economic or other reasons, thus ensuring product compliance at a specified level of confidence, or conversely, at an acceptable level of risk. In a binary decision rule, stringent acceptance is accompanied by relaxed rejection. Relaxed rejection allows the rejection of products even when the measurement result lies within the specification zone by the guard band amount. The size of the guard band is expressed as a percentage of the expanded uncertainty. It is typically the customer who requests stringent acceptance of the supplier and enforces this through the contract. Some of the factors that should be considered when establishing the size of the guard band are given in Appendix E.

Figure 3 illustrates examples of stringent acceptance/relaxed rejection. The guard band applied within the specification zone,  $g_{In}$ , usually is determined by estab-

lishing an "acceptable risk" of accepting out-of-specification products. One-sided stringent acceptance is used to guard band only one of the specification limits. For example, workpiece "form error" is always positive by definition, hence the lower limit (zero) does not require a guard band. Measurement results that lie in the acceptance zone are considered to verify the product to its specification.

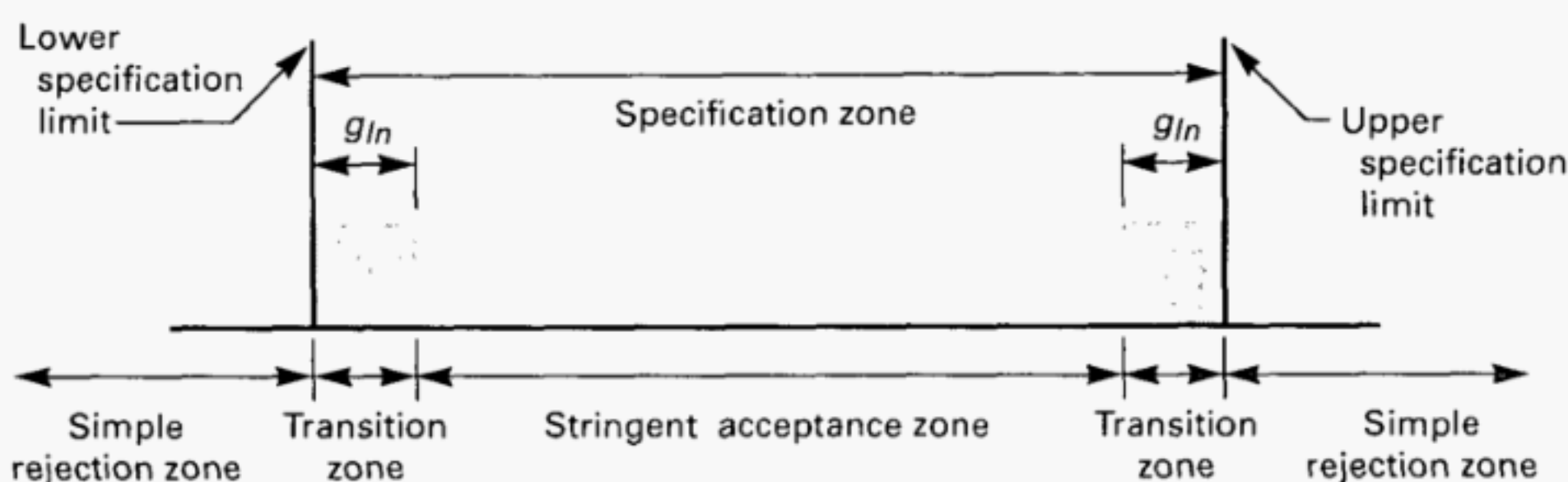
#### 4.3 Stringent Rejection and Relaxed Acceptance Using a Z% Guard Band

Stringent rejection increases confidence that a rejected product is actually out-of-specification. Adding the guard band amount(s) to the specification zone creates the rejection zone. It is typically the supplier who requests stringent rejection of the customer who may be seeking a refund for a product that is claimed to be out-of-specification. In a binary decision rule stringent rejection is accompanied by relaxed acceptance. Relaxed acceptance allows acceptance of products with measurement results that lie outside the specification zone by the guard band amount. Relaxed acceptance is often used when a state-of-the-art measurement system still has such large uncertainty that a significant number of



GENERAL NOTE: Products are rejected if the measurement result is within the rejection zone.

**FIG. 4 SYMMETRIC TWO-SIDED RELAXED ACCEPTANCE AND STRINGENT REJECTION**



GENERAL NOTE: Products are accepted if the measurement result is within the acceptance zone, rejected if in the rejection zone, and subject to a different rule in the transition zone.

**FIG. 5 STRINGENT ACCEPTANCE, SIMPLE REJECTION, AND A TRANSITION ZONE EXAMPLE USING SYMMETRIC TWO-SIDED GUARD BANDING**

good products would be rejected under simple or stringent acceptance rules. Figure 4 is an example of a binary decision rule using relaxed acceptance.

#### 4.4 Decision Rules With a Transition Zone

In some measurement situations additional alternatives to acceptance or rejection may be desirable. These can be implemented by the use of transition zones that lie in between the acceptance and rejection zones. The location and decision outcome of any transition zones must be documented in the decision rule.<sup>22</sup> Figure 5 presents an example of stringent acceptance, simple rejection, and a transition zone created by symmetric two-sided  $Z\%$  guard banding. An example of a decision outcome for a measurement result in the transition zone is the acceptance of the product at a reduced price.

<sup>22</sup> It is crucial that both the supplier and customer agree upon both the size of the guard band and the decision outcome for a measurement result occurring in this zone; lack of these agreements may lead to costly negotiations and legal expenses.

## 5 EXAMPLES OF DECISION RULES

A decision rule must fulfill the requirements of section 3. Hence the concepts of simple, stringent, or relaxed acceptance or rejection discussed in section 4 need elaboration in order to become decision rules. Some examples of complete decision rules are given in paras. 5.1–5.4. For binary decision rules a shorthand name appears in parentheses describing the acceptance properties (since rejection can be deduced).

### 5.1

(Simple 4:1 Acceptance)

Simple Acceptance Using a 4:1 Ratio with Mean Measurement Results and Rejection with Cause.<sup>23</sup>

<sup>23</sup> See Fig. 2.

**5.2****(100% Stringent Acceptance)**

Stringent acceptance and relaxed rejection using symmetric 100% two-sided guard bands with mean measurement results and rejection with cause.<sup>24</sup>

**5.3****(100% Relaxed Acceptance)**

Relaxed acceptance and stringent rejection using symmetric 100% two-sided guard bands with mean measurement results and rejection with cause.<sup>25</sup>

**5.4**

**(Stringent Acceptance and Simple Rejection Using Symmetric 50% Two-Sided Guard Bands with Mean-Measurement Results and Rejection with Cause)**

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<sup>24</sup> See Fig. 3 (first drawing).

Transition zone outcome is 20% price reduction for measurement results in these zones.<sup>26</sup>

In the examples given in 5.1–5.4, if repeated measurements are performed then the mean result is used to verify acceptance.<sup>27</sup> The uncertainty of the mean of multiple measurements may be less than the uncertainty of a single measurement. Similarly, measurements can only be rejected if a physical effect is identified as the cause for the spurious result.

Appendix F briefly discusses the ISO 14253-1 standard as it pertains to decision rules.

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<sup>25</sup> See Fig. 4.

<sup>26</sup> See Fig. 5.

<sup>27</sup> Using the mean of several measurement results is appropriate for workpiece characteristics. For some instrument specifications the mean measurement result may be inappropriate. See Appendix B.

## NONMANDATORY APPENDIX A

### APPLICATION OF DECISION RULES IN THE CUSTOMER-SUPPLIER RELATIONSHIP

The choice of a decision rule is ultimately a business decision. It includes such factors as

- (a) the cost of rejecting an in-specification product;
- (b) the cost of accepting an out-of-specification product;
- (c) uncertainty associated with the measurement process;
- (d) the distribution of the product's characteristic under consideration; and
- (e) the cost of making measurements.

Once a decision rule is formulated, the responsibility for its application should be unambiguously defined, in particular, which party (customer or supplier) will apply a particular rule. For example, the use of stringent acceptance with a 100% guard band may be a reasonable requirement on the supplier if their measurement uncertainty is small relative to the specification zone. On the other hand, the same decision rule used by a customer having a large measurement uncertainty relative to the specification zone could result in very few products being accepted.<sup>1</sup> Since there are obvious economic consequences associated with the use of

decision rules and which party employs them, this issue should be resolved in the contract negotiations. The negotiated price of the product may vary significantly depending on which party applies which decision rule, the uncertainty of the measurements, and the required level of confidence.

In some contractual situations different decision rules may be used for the supplier and the customer, e.g., see International Standard 14253-1. For example, a supplier may be required to use a decision rule involving stringent acceptance and relaxed rejection in order to sell the product to the customer. The same contract may require the customer to use stringent rejection and relaxed acceptance in order to demonstrate that the product is out-of-specification. In this example, there is an additional burden on the supplier (i.e., stringent acceptance) before they can sell the product, similarly there is an additional burden on the customer (i.e., stringent rejection) before they can reject a product. The use of this contract in this situation should greatly reduce any conflict regarding the acceptance or rejection of the product. If conflict still exists, e.g., the supplier demonstrates acceptance and the customer demonstrates rejection, then a first step in a resolution could be to examine the reliability of each party's uncertainty statement. This issue is considered in ASME B89.7.3.3 (in the course of preparation).

<sup>1</sup> A large uncertainty relative to the specification zone may be an indicator of inappropriate measurement equipment; the 100% guard band protects against accepting potentially out-of-specification products with a significant economic cost.

## NONMANDATORY APPENDIX B REPEATED MEASUREMENT

### B1 WORKPIECES

It is not uncommon for workpiece inspectors to repeat measurements, particularly if the measurement result lies just outside the acceptance zone. A subsequent measurement may lie within the acceptance zone leading to a dilemma regarding the status of the product. Ad-hoc procedures, such as selecting the best two out of three measurement results (see Youden) or rejecting measurements deviating more than three or four standard deviations from the mean, are unreliable, hence an alternative procedure is needed. A conservative approach appropriate for workpiece characteristics is to use the mean of the measurement results as the best estimate of the product characteristic under inspection. If the mean result lies in the acceptance zone then the product can be considered acceptable. Measurement results cannot be rejected simply because they produce undesirable results, and measurements should be rejected only if it clearly can be shown that the result was spurious. See Appendix C.

Depending on the details of the measurement process, it may be possible to reduce the combined standard uncertainty with the use of repeated measurements. Hence in the case of stringent acceptance, the guard band,  $g_{ln}$ , might be reduced in magnitude. Many uncertainty budgets for workpiece characteristics are developed for a single measurement result; if significant uncertainty contributors are present that arise from independent random variables,<sup>1</sup> e.g., uncertainty due to

repeatability, then the standard uncertainty for these sources is expected to decrease when using the mean of several measurement results. For these sources, the standard uncertainty of these contributors will typically decrease inversely with the square root of the number of measurements. Hence a new, somewhat smaller combined standard uncertainty may be calculated, resulting in a smaller guard band,  $g_{ln}$ . If the mean of the repeated measurements lies within this enlarged acceptance zone the product can be accepted according to the decision rules.

### B2 INSTRUMENTS

With respect to the testing of instrumentation, the number of repeated measurements that may be performed during a performance test is controlled and often repeated measurements are not allowed as the instrument reproducibility is one of the characteristics under investigation by the test. Accordingly, repeated measurements during instrument performance tests are generally not allowed unless explicitly permitted in the testing procedure. For example, suppose the acceptance test for a caliper is to measure a calibrated gauge block ten times and determine that the largest observed error is less than the supplier's stated MPE appropriated for a single measurement. If the caliper's errors are randomly distributed (i.e., no systematic errors) then some will be positive (i.e., the block is measured too long) and some will be negative (i.e., the block is measured too short). Since the measurand of interest in this test is the error of a single reading, no averaging of errors is permitted.

Whatever method is chosen for addressing repeated measurements, it must be clearly defined and referred to when defining a decision rule.

<sup>1</sup> Often these sources will be "Type A" uncertainty sources as designated in the GUM; however, "Type B" uncertainties sometimes represent independent and random uncertainty sources and consequently will also be reduced in magnitude as a result of repeated measurements. An example of a "Type B" uncertainty that may represent a random uncertainty source is a "repeatability specification" provided on an instrument specification sheet of the instrument that is used to inspect a workpiece characteristic.



## NONMANDATORY APPENDIX D

### SPECIAL ISSUES OF DECISION RULES FOR INSTRUMENTATION

The specification of instruments typically is in terms of a maximum permissible error (MPE), i.e., the largest observed error (regardless of sign) from a test procedure must be less than a supplier-specified MPE. Hence the specification zone has a width of twice the MPE, i.e.,  $\pm\text{MPE}$  and an  $N:1$  decision rule require the uncertainty interval (of width  $2U$ ) to be no greater than  $1/N$  of this value, hence the expanded uncertainty,  $U$ , is to be no greater than  $1/N$  of the MPE value. (In contrast, with a true one-sided guard band, as occurs with workpiece form errors, an  $N:1$  decision rule requires the expanded uncertainty to be  $1/(2N)$  of the specification value.)

An instrument specification is often the result of some type of evaluation test; hence the specification zone is in units of the test result.<sup>1</sup> Similarly, the guard band values are calculated in terms of the test uncertainty. To determine the uncertainty in the test result, first the uncertainty associated with each of the individual test measurements must be determined. Then these uncertainties must be propagated through the test analysis; this may weight the uncertainty of some measurements more significantly than others, e.g., as in a RMS-formulated test result.

The uncertainty of an individual test measurement is just the root-sum-of-squares (RSS) of the uncertainty in the display resolution of the instrument and the uncertainty in the realization of the measurand embodied by the calibrated standard in use. The standard is intended to represent a "true value" of the measurand. Unfortunately, all standards have an associated uncertainty. This includes the uncertainty documented in its report of calibration and the uncertainty in the standard due to the conditions at the time it is used as a reference standard. Uncertainty sources associated with the conditions of use include thermal effects, clamping distortions, contamination, and similar problems that degrade the accuracy of the standard. The combined uncertainty due to both the calibration report uncertainty and the uncertainty due to the conditions at the time of use,

is described as "uncertainty in realizing the measurand" of the standard (see Phillips, et al.). It is this uncertainty, combined in an RSS manner with the uncertainty in the instrument's resolution, that is propagated through the test analysis to yield the final test uncertainty.

It is sometimes erroneously believed that the systematic error and reproducibility of the instrument under test are also to be included in the uncertainty analysis. This is incorrect, as the systematic error and reproducibility are the metrological characteristics under examination by the performance test. It is only the RSS of the uncertainty in the instrument's resolution and the uncertainty in the realization of the measurand of the standard used in the testing procedure that must be considered by a decision rule.

The calculation of the guard bands proceeds similarly to the general case discussed in Appendix E, i.e.,  $g_{In} = h_{In} U$ , where  $U$  is the expanded uncertainty of the test result. In the case of instrumentation, the optimal value of the guard band is typically larger than in the workpiece case, since errors associated with an instrument will typically propagate into a large number of subsequent measurements performed using the instrument, so economic considerations will tend to increase the magnitude of the guard band.

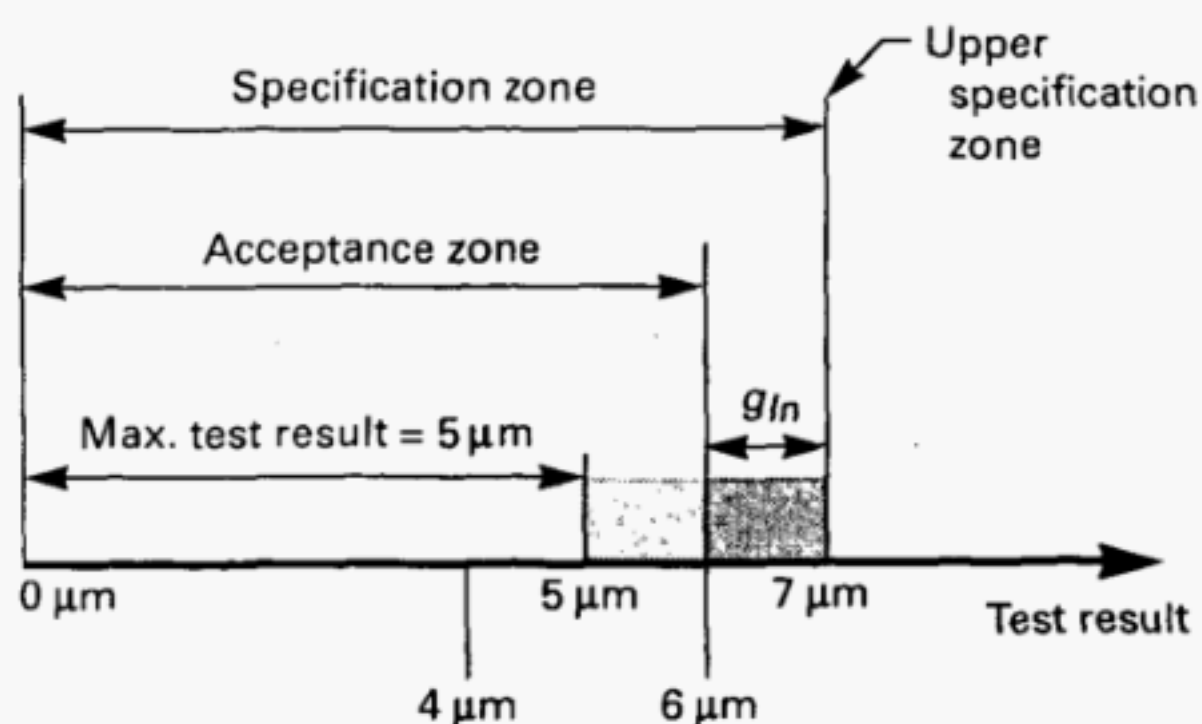
When verifying the specifications of an instrument using the guard banding approach, it is crucial that the customer use a reference standard with an uncertainty no greater than that prescribed by the supplier. Similarly when a supplier establishes an instrument's specification limits, the uncertainty of the realization of the measurand of the standard will play an important role in the economics of establishing the specification limit. From this perspective, it is instructive to consider how the uncertainty of the standard becomes incorporated into the specification limit.

Consider a supplier who seeks to establish a specification limit on an instrument according to a given performance test. (Assume for simplicity, that the test procedure is just to select the largest of a series of observed errors in the measurement of a calibrated standard.) The supplier uses a calibrated standard and tests several instruments and observes that the worst-

<sup>1</sup> The units of the test result need not be the same as those of a measurement result; for example, the test may report the sum-of-squares of the measurement errors.

case test result is less than  $5\text{ }\mu\text{m}$ . Given this information, and knowledge of the uncertainty of the standard, what specification limit should be assigned so that all of the product will be verified to be in specification according to the guard banding decision rule? Suppose that the calibrated standard has an uncertainty that propagates to an uncertainty in the test results of  $1\text{ }\mu\text{m}$ . Hence, when using another nominally identical standard having the same uncertainty the worst-case test results reasonably could range between  $4\text{ }\mu\text{m}$  and  $6\text{ }\mu\text{m}$ . See Fig. D1.

Suppose further that the contract with the customer specifies that stringent acceptance with a 100% guard band will be used in the acceptance testing of the instrument. Consequently the specification limit must be set to  $7\text{ }\mu\text{m}$  so that an acceptance zone of  $6\text{ }\mu\text{m}$  allows the sale of the instruments when tested with a calibrated standard that results in no more than  $1\text{ }\mu\text{m}$  uncertainty in the test results. This illustrates the general principle that the specification limit must be set equal to the largest test result plus twice the test uncertainty ( $5\text{ }\mu\text{m} + 2\text{ }\mu\text{m} \times 1\text{ }\mu\text{m} = 7\text{ }\mu\text{m}$ ) in order to have



**FIG. D1 AN EXAMPLE OF CREATING THE SPECIFICATION LIMIT TAKING INTO ACCOUNT TEST RESULTS AND TEST UNCERTAINTY**

a high probability of selling the product. Accordingly, the use of well-calibrated standards in both the establishment of the specification limits and the subsequent acceptance testing is a very significant consideration.

## NONMANDATORY APPENDIX E DETERMINATION OF GUARD BAND LIMITS

Calculation of a guard band,  $g_{In}$ , used in stringent acceptance and applied within the specification zone, typically starts with the calculation of the expanded uncertainty,  $U$ . This is a quantitative measure of the uncertainty of the measurement based on metrological considerations. The value of  $g_{In}$  depends strongly on the product being considered and is influenced by economic factors. This can be expressed by the relation  $g_{In} = h_{In} U$ , where  $h_{In}$  includes the economic factors, some of which are described in Appendix A. For clarity,  $g_{In}$  is usually stated as a percentage, i.e.,  $h_{In} = 1$  yields a guard band whose width is equal to 100% of the expanded uncertainty.

Consequently,  $g_{In}$  depends on metrological (quantified by  $U$ ) and economic (quantified by  $h_{In}$ ) issues. If the relative cost of accepting an out-of-specification workpiece is low, i.e.,  $h_{In}$  is small or zero, then  $g_{In}$  will be small or zero for any reasonable value of  $U$ . Conversely, if the cost of accepting an out-of-specification product is high, then  $g_{In}$  will be large (see Williams and Hawkins). In some situations where the cost of

the workpiece is high, the uncertainty interval is comparable to the specification zone, and the cost of accepting an out-of-specification workpiece is low, then the economics may favor relaxed acceptance where  $h_{Out}$  is large, e.g.,  $h_{Out} = 1$ . In this example the guard band  $g_{Out}$  is large, e.g., 100% of the expanded uncertainty.

An equivalent way of interpreting  $h$  is to establish the acceptable probabilities of pass and fail errors, based on economic considerations, as described in Appendix D of ASME B89.7.2-1999. Similarly, while ASME B89.7.3.1 emphasizes the widths of the various zones (analogous to ISO 14253-1), ASME B89.7.2 emphasizes the limits of the acceptance zone, known as gauging limits.

The calculation of the guard band applied outside the specification zone,  $g_{Out}$ , typically is based on the level of confidence needed to reject the product. Hence sufficient confidence should be established that rejecting the product will withstand (often legal) scrutiny. In general the guard band used in stringent acceptance will have a different magnitude than the guard band used in stringent rejection.

## NONMANDATORY APPENDIX F

### A DISCUSSION OF ISO 14253-1

ISO 14253-1 attempts to define a set of default decision rules. The standard specifies that the supplier of a product is to use stringent acceptance in order to sell the product. The customer of the product, using their own measurement uncertainty, similarly uses a 100% guard band in stringent rejection.

The default rules require the supplier to verify stringent acceptance in order to sell the product, and the customer to demonstrate stringent rejection in order to reject the product. No information is supplied regarding the decision outcome if the supplier's measurement result lies in their transition zone. Similarly, no informa-

tion is supplied regarding the use of repeated measurements or the rejection of outliers. Also, there is no consideration of economic factors in the default rule.

The absence of a decision outcome for measurement results that lie in the transition zone is particularly troublesome for customers who become resellers of the product. Contrary to the claim in 14253-1 that this situation only occurs when the reseller's uncertainty is larger than the supplier's, it is a likely outcome even when the reseller's uncertainty is smaller than the supplier's as shown in Fig. F1.

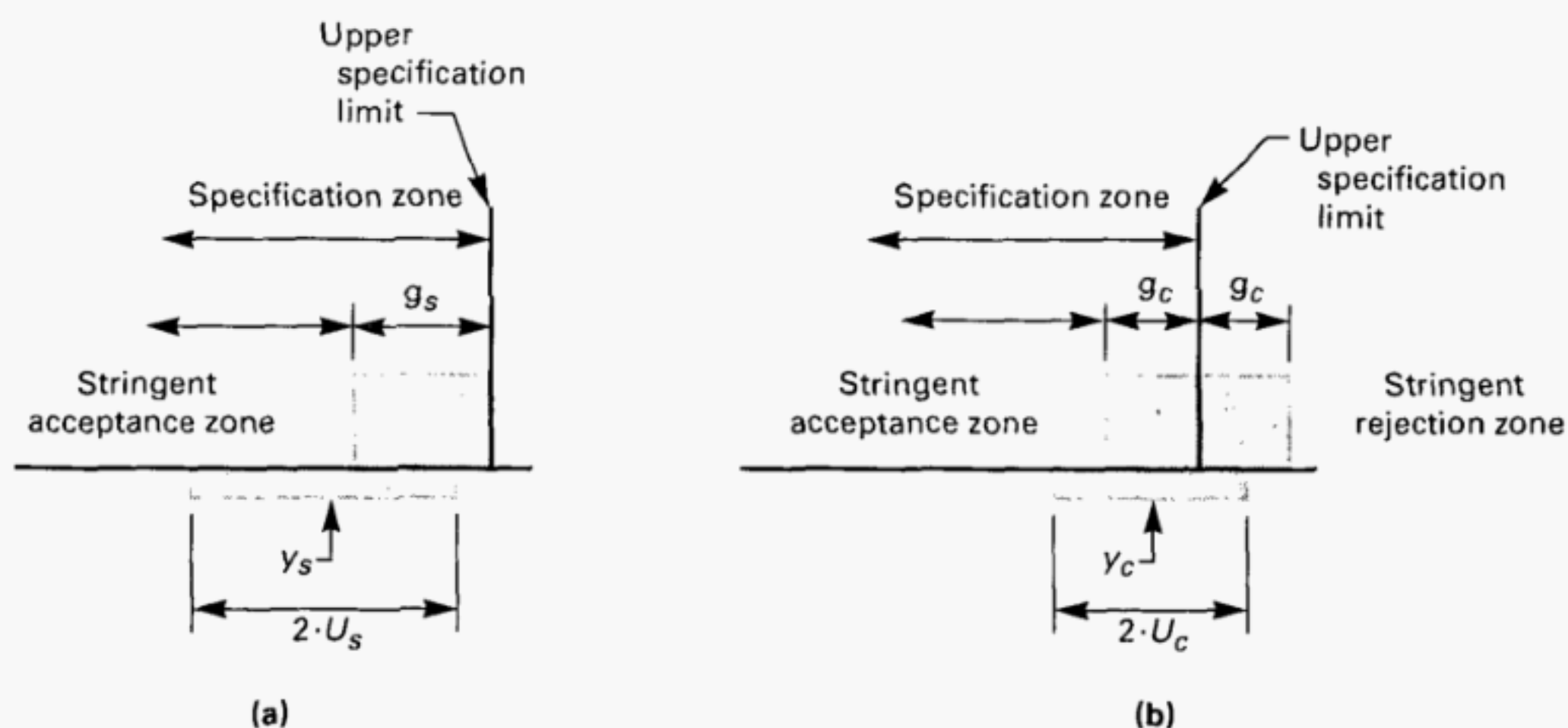


FIG. F1 EXAMPLES RELATING TO ISO 14253-1

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