

**ASME B89.7.2-2014**

**[Revision of ASME B89.7.2-1999 (R2004)]**

# **Dimensional Measurement Planning**

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**AN AMERICAN NATIONAL STANDARD**



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Mechanical Engineers**

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[Revision of ASME B89.7.2-1999 (R2004)]

# Dimensional Measurement Planning

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**The American Society of  
Mechanical Engineers**

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# FOREWORD

The intent of this Standard is to facilitate agreement between suppliers and customers by specifying a standard method for assessing the dimensional acceptability of workpieces. Components of the method are the preparation of an adequate dimensional measurement plan and the use of the plan in making measurements. Major inputs to the method are dimensional specifications developed, for example, in compliance with ASME Y14.5-2009, Dimensioning and Tolerancing [1].

The first publication of ASME Y14.5 was a significant step forward in manufacturing because it defined methods for the unambiguous expression of design intent on workpiece drawings. ASME Y14.5 specifies design intent in terms of workpiece features (e.g., cylinders, planes, spheres, etc.). A feature is dimensioned and toleranced by specifying boundaries within which the infinite number of points on the feature surface must lie (for surface geometries) or by specifying a boundary within which the axes must lie (axes control). Any adequate assessment of whether a manufactured feature complies with an ASME Y14.5 drawing specification must consider this infinite number of points.

In the early days of ASME Y14.5, serious attempts to determine compliance of workpiece features with drawing specifications were based on gaging by attributes, e.g., by means of ring gages, plug gages, and functional gages. Such gages dealt with the infinite number of points by means of gaging surfaces, which were intended as the ideal counterparts of the surfaces to be measured. Uncertainty due to gage imperfections was minimized by specifying gages whose probable errors were tightly controlled when compared with the tolerances of the workpieces to be measured. Gage dimensions were biased to ensure that no nonconforming workpieces were accepted, even though such biasing resulted in the rejection of some conforming workpieces.

In 1994, a supporting standard was released that explicitly defined the mathematical expression of ASME Y14.5, which was ASME Y14.5.1M-1994, Mathematical Definition of Dimensioning and Tolerancing Principles [7]. This Standard presents a mathematical definition of geometrical dimensioning and tolerancing consistent with the principles and practices of ASME Y14.5M-1994, enabling determination of actual values.

Since the capability of a typical manufacturing process has improved more rapidly than the measurement capability associated with gaging by attributes, the old methods have led to expensive increases in the rejection of conforming workpieces. Statistical analysis capabilities and cost effectiveness have led to the proliferation of coordinate-measuring machines (CMMs) that cannot directly verify dimensional acceptability using an infinite number of points in a workpiece feature surface but account for this in the associated measurement uncertainty statement. In some instances, the algorithms used to associate substitute geometrical elements according to drawing specifications result in significant measurement uncertainty. ASME Working Group B89.3.2 (now B89.7.2) was formed to address these and related issues.

One of these issues is the criterion for acceptable dimensional measurement practice. A measurement process should be designed to balance measurement quality and cost, including costs associated with decision outcomes resulting in rejecting conforming products or accepting nonconforming products due to the measurement uncertainty. While the analysis of costs is workpiece-specific and outside the scope of this dimensional measurement Standard, the measurement process should be designed to provide the required metrological data for the risk analysis needed to formulate a decision rule.

Measurement quality is characterized in terms of measurement uncertainty. Previous practice has been to assume that gage quality was controlled to a level where the contribution to measurement uncertainty due to gaging error was negligible. This assumption was applied both to measurement by attributes, as described above, and to measurement by variables using simple bench tools such as micrometers and height gages. Gage repeatability and reproducibility (GR&R) studies provide useful information relating to uncertainty but they cannot, in themselves, completely determine measurement uncertainty values.

The Guide to the Expression of Uncertainty in Measurement (GUM) [2] and the equivalent U.S. standard, ANSI/NCSL Z540.2-1997 [4], are considered to be the authoritative documents on the evaluation of measurement uncertainty. A recent supplement to the GUM, JCGM 101:2008 [5], describes the use of Monte Carlo methods for uncertainty evaluation.

The ASME B89.7 Subcommittee has developed a series of standards and technical reports pertaining to the evaluation of measurement uncertainty, decision rules and conformity assessment, and metrological traceability considerations. These documents include

- B89.7.3.1, Guidelines for Decision Rules: Considering Measurement Uncertainty in Determining Conformance to Specifications
- B89.7.3.2, Guidelines for the Evaluation of Dimensional Measurement Uncertainty





- B89.7.3.3, Guidelines for Assessing the Reliability of Dimensional Measurement Uncertainty Statements
- B89.7.4.1, Measurement Uncertainty and Conformance Testing: Risk Analysis
- B89.7.5, Metrological Traceability of Dimensional Measurements to the SI Unit of Length

The ASME B89.7.2 Standard makes use of the methods of the foregoing documents for the evaluation of measurement uncertainty, formulation of decision rules, calculation of the risks of mistaken decisions, and, when desired, demonstration of metrological traceability to the SI unit of length, the meter.

In considering its assignment, the ASME B89.7.2 Working Group determined that a single “cookbook” standard covering all valid methods for measuring all possible workpiece features for all possible purposes under all possible conditions would be impractical. Among the problems are the difficulty of writing and maintaining such an extensive document, lack of documentation for some types of measurements, and rapidly changing technology.

The approach of the current Standard is to identify the principles applicable to all dimensional measurements, and to cite detailed standards for specific classes of measurements as they become available. Two strategies are used. The first is to ensure the validity of dimensional measurements by specifying requirements for preparation, approval, and use of dimensional measurement plans. The second is to provide appendices that discuss methods and resources for developing such plans.

The ASME B89.7.2 Standard considers that a measurement method is acceptable if it results in an acceptable measurement uncertainty. Thus, for example, a gage producing a limited point data set (e.g., a CMM) may be used to determine compliance with ASME Y14.5 if the uncertainty component due to the limited data can be reasonably evaluated and if the resultant combined standard uncertainty is acceptable according to the decision rule and the target uncertainty. The decision rule and target uncertainty is determined by management and is an appropriate balance between measurement quality and cost. For example, if a manufacturing process produces few nonconforming workpieces, and the impact of an out-of-tolerance workpiece is low, then a low-accuracy measurement method may be adequate. For workpieces where an out-of-tolerance condition could cause serious injury and the cost of rejecting a conforming workpiece is high, the measurement requirement might be stringent and the acceptable measurement uncertainty small. Such considerations may be embodied in contracts or company policies.

The body of this Standard delineates requirements and recommendations for dimensional measurement planning. Actions required for compliance with the Standard are identified by use of the word “shall.” Compliance with other identified actions is strongly recommended to ensure quality in measurement. The appendices provide examples of how to develop a plan, how to select gaging, and how to evaluate various components of measurement uncertainty. Means are presented for determining the probabilities of decision outcomes in workpiece acceptance or rejection. Such probabilities are useful in evaluating plan acceptability. A reference section is also included. The Standard provides the user with means for meeting the requirements of ANSI/ASQC E2, Guide to Inspection Planning [6].

It is anticipated that future work of the ASME B89.7.2 Working Group will be in the area of updating and revising this second edition of the Standard in response to further study, public comments, and other standards developments.

The first edition of this Standard was approved by the American National Standards Institute (ANSI) on October 26, 1999. This 2014 edition of ASME B89.7.2 was approved by ANSI as an American National Standard on July 17, 2014.





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The Committee welcomes proposals for revisions to this Standard. Such proposals should be as specific as possible, citing the paragraph number(s), the proposed wording, and a detailed description of the reasons for the proposal, including any pertinent documentation.

When appropriate, proposals should be submitted using the B89 Project Initiation Request Form.

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Subject: Cite the applicable paragraph number(s) and the topic of the inquiry.  
Edition: Cite the applicable edition of the Standard for which the interpretation is being requested.  
Question: Phrase the question as a request for an interpretation of a specific requirement suitable for general understanding and use, not as a request for an approval of a proprietary design or situation. The inquirer may also include any plans or drawings that are necessary to explain the question; however, they should not contain proprietary names or information.

Requests that are not in this format may be rewritten in the appropriate format by the Committee prior to being answered, which may inadvertently change the intent of the original request.

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# DIMENSIONAL MEASUREMENT PLANNING

## 1 SCOPE

### 1.1 Objective

The objective of this Standard is to ensure correctness and acceptability of dimensional measurements.

### 1.2 Requirements

This Standard specifies requirements for preparation and approval of dimensional measurement plans and for the use of approved plans in making dimensional measurements.

### 1.3 Applicability

This Standard considers that a dimensional measurement method is acceptable if its associated measurement uncertainty per the Guide to the Expression of Uncertainty in Measurement (GUM) [2] meets business needs, e.g., cost of measurements, consequences of pass and fail errors, liability, specific policies, and customer requirements. In assessing the acceptability of a stated measuring procedure, management should take into account the importance and difficulty of evaluating measurement uncertainty in such a way as to quantitatively capture the effects of all significant sources of measurement error.

### 1.4 Purpose

This Standard is intended for use by process and quality engineers or personnel performing a similar function in planning dimensional measurements.

## 2 DEFINITIONS

*accept-reject measurement*: measurement made for the purpose of accepting or rejecting a workpiece or workpiece feature or property.

*attribute gage*: gage that verifies conformance of a workpiece property with a specified requirement without yielding a measured value.

*conforming*: having a true value lying within or on the boundary of a stated tolerance interval.

NOTE: The issue of conformance/nonconformance involves where the true value of the measurand lies with respect to the tolerance zone; the issue of acceptance/rejection involves where a measured value lies with respect to the acceptance zone.

*consumer's risk*: probability of a pass error.

*dimensional measurement*: measurement of a geometrical property of a workpiece.

*dimensional measurement plan*: plan detailing equipment, environment, and procedure for measuring one or more geometrical properties of a workpiece or for measuring the properties of a process.

*dimensional measurement planner (DMP)*: person who prepares a dimensional measurement plan.

*fail error*: rejection, as a result of measurement uncertainty, of a conforming property of a process or a workpiece.

NOTE: A fail error is also known as false rejection or a Type 1 error.

*failure mode and effects analysis (FMEA)*: for a system, the steps of listing failure modes and determining effects of failure on the system in each mode.

*feature*: general term applied to physical portion of a workpiece, such as a surface, hole, or slot.

*gaging limits*: specified limits of a measured value of a workpiece property.

NOTE: Gaging limits are also known as acceptance limits.

*geometric characteristic*: dimensional description of a feature.

*manufacturing plan*: plan detailing organization, equipment, environment, personnel, and procedures for manufacturing a workpiece.

*measurand*: quantity intended to be measured.

*nonconforming*: having the true value lying outside the boundaries of a stated tolerance interval.



NOTE: The issue of conformance/nonconformance involves where the true value of the measurand lies with respect to the tolerance zone; the issue of acceptance/rejection involves where a measured value lies with respect to the acceptance zone.

*pass error*: acceptance, as a result of measurement uncertainty, of a nonconforming property of a process or a workpiece.

NOTE: A pass error is also known as false acceptance or a Type 2 error.

*process control measurement*: measurement of the features or properties of sampled workpieces in order to detect a change in the process.

*process failure*: change or changes in a process resulting in production of unacceptable workpieces.

*producer's risk*: probability of a fail error.

*tolerance*: total amount a specific dimension is permitted to vary. The tolerance is the difference between the maximum and minimum limits.

NOTE: The tolerance interval is the region between and including the tolerance limits

*uncertainty (of measurement)*: non-negative parameter characterizing the dispersion of the values being attributed to a measurand, based on the information used.

NOTE: Uncertainty is different from the measurement error, which is the difference between the measured value and the true value.

### 3 NORMATIVE REFERENCES

ASME B89.7.3.1-2001, Guidelines for Decision Rules: Considering Measurement Uncertainty in Determining Conformance to Specifications

ASME B89.7.3.2-2007, Guidelines for the Evaluation of Dimensional Measurement Uncertainty

ASME B89.7.3.3-2002, Guidelines for Assessing the Reliability of Dimensional Measurement Uncertainty Statements

ASME B89.7.4.1-2005, Measurement Uncertainty and Conformance Testing: Risk Analysis

ASME B89.7.5-2006, Metrological Traceability of Dimensional Measurements to the SI Unit of Length

ASME Y14.5-2009, Dimensioning and Tolerancing

ASME Y14.5.1M-1994, Mathematical Definition of Dimensioning and Tolerancing Principles

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JCGM 100:2008, Evaluation of measurement data — Guide to the expression of uncertainty in measurement (GUM)

JCGM 200:2012, International vocabulary of metrology — Basic and general concepts and associated terms (VIM)

Publisher: Joint Committee for Guides in Metrology, Bureau International des Poids et Mesures, Pavillon de Breteuil, F-92312 Sèvres Cedex, France ([www.bipm.org/en/publications/guides](http://www.bipm.org/en/publications/guides))

## 4 DIMENSIONAL MEASUREMENT PLANNING

### 4.1 General

For each workpiece to be measured, a dimensional measurement plan shall determine and define necessary measurement tasks and shall specify their requirements.

(a) Necessary measurement tasks can be process control measurements, accept-reject measurements, or both.

(b) Each process control measurement should relate to a controllable parameter of the workpiece manufacturing process.

### 4.2 Prerequisites

**4.2.1 Dimensional Measurement Planner.** The Dimensional Measurement Planner (DMP) should have the requisite knowledge for adequate consideration of all applicable aspects of the dimensional measurement plan (e.g., dimensioning and tolerancing specifications, statistics, sampling plans, manufacturing plans, measurement uncertainty evaluation, etc.).

**4.2.2 Drawings and Specifications.** The DMP shall review workpiece drawings and specifications to ensure dimensional requirements are clear, fully defined, and understood. The measurands chosen to establish a workpiece's conformance to the dimensional requirements rely on the clarity and completeness of this information. Where these conditions are not met, the DMP shall contact those responsible for the drawings and specifications and obtain documented clarification, in accordance with specified standards and revision levels.

Participation in design reviews with designers can help the DMP to gain an understanding of the true design intent. Such insight into design intent aids the DMP who is responsible for ensuring that the specified requirements are not only valid but also represent the true design intent. This emphasis is given to ensure that drawing specifications are not overconstrained in such a way as to allow workpieces to be unnecessarily rejected when they will meet all functional requirements. Workpiece drawings and specifications define the measurands to be measured.

**4.2.3 Process Characteristics.** Where applicable, the DMP shall review the manufacturing plan and manufacturing process failure mode and effects analysis to determine process tendencies and requirements for prediction of potential process failure. The DMP shall work with the manufacturing engineer to determine process control measurement requirements.





#### 4.2.4 Dimensional Measurement Planning Constraints.

The DMP shall review acceptable probabilities of pass errors and fail errors, and estimated costs of inspection and equipment.

**4.2.5 Existing Equipment.** The DMP shall review the inventory of available gages and equipment and determine adequacy for ensuring that measurement requirements are fully met.

### 4.3 Dimensional Measurement Plan

The dimensional measurement plan shall contain or reference all information needed for making measurements. Where applicable, the following considerations shall be addressed in developing the dimensional measurement plan (see Nonmandatory Appendix A for an example):

- (a) workpiece identification (e.g., workpiece number, revision level, serial number, description)
- (b) determination of measurands (features to be measured from workpiece drawing, process sheet, inspection instructions, etc.)
- (c) determination of measurements to be taken (e.g., for process control, workpiece acceptance, etc.)
- (d) determination of lot sampling plan and sample analysis methods (sample size, frequency)
- (e) determination of how measurements will be taken (e.g., on-line or off-line, indicating instruments or gages, automatic or manual)
- (f) determination of measurement strategy (e.g., number and location of measured points, sequence of operations, etc.)
- (g) determination of report documentation and archival requirements (e.g., paper report, electronic data files, duration of record maintenance, etc.) and retention of raw measurement data
- (h) determination of documentation requirements for metrological traceability of dimensional measurements per ASME B89.7.5 (e.g., ISO/IEC 17025 calibration reports, documented company quality assurance policy, etc.)
- (i) determination of data evaluation requirements (e.g., mathematical model, association criteria, algorithms, filters, cutoffs, filter parameters, graphical outputs, etc.)
- (j) selection of candidate gage for each measurement (e.g., coordinate-measuring machine, special fixturing, handheld gages, analytical equipment, etc.)

NOTE: See Nonmandatory Appendix B for more information.

- (k) determination of required measurement skill and time availability
- (l) determination of location(s) in plant where measurement(s) will be taken (e.g., workstation, inspection bench, laboratory, etc.) and gage or workpiece portability, ergonomics, workstation-related factors and environmental effects
- (m) determination of measurement cycle time versus available time

(n) determination of any special user instructions and documentation requirements (e.g., setup orientation, fixturing, clamping, probing strategy, operator's manuals, etc.)

(o) evaluation of the measurement uncertainty;

(p) calculation of the probabilities of pass errors and fail errors for accept-reject measurements, based on the probability distributions associated with the manufacturing process and the measuring system

NOTE: See Nonmandatory Appendix D for more information.

(q) gaging limits consistent with the specification and the probabilities of pass and fail errors

(r) determination of the disposition of workpieces (e.g., quarantine nonconforming workpieces, etc.)

(s) review suitability of the dimensional measurement plan and adjust as needed to meet business requirements (e.g., probabilities of pass errors and fail errors, reasonableness, practicality, applicability of results, etc.)

## 5 DIMENSIONAL MEASUREMENT PLAN APPROVAL

The dimensional measurement plan, and any subsequent changes to the plan, shall be approved as required (e.g., by company policy or by contract, etc.).

## 6 DIMENSIONAL MEASUREMENT PLAN APPLICATION

### 6.1 Dimensional Measurement Requirements

Dimensional measurements shall be performed as specified in the dimensional measurement plan.

### 6.2 Dimensional Measurement Plan Revision

The DMP shall update the dimensional measurement plan in response to changes in workpiece specification, the manufacturing process, and/or the measurement process.

## 7 REFERENCES

This list of references is supplementary to those given in section 3 of this Standard. They may be useful in understanding dimensional measurements and in writing dimensional measurement plans.

- [1] ASME Y14.5-2009, Dimensioning and Tolerancing.
- [2] JCGM 100:2008, Evaluation of measurement data — Guide to the expression of uncertainty in measurement (GUM).
- [3] JCGM 200:2012, International vocabulary of metrology — Basic and general concepts and associated terms (VIM).
- [4] ANSI/NC SL Z540.2-1997 (R2012), U.S. Guide to the Expression of Uncertainty in Measurement.
- [5] JCGM 101:2008, Evaluation of measurement data — Supplement 1 to the "Guide to the expression of





- uncertainty in measurement" — Propagation of distributions using a Monte Carlo method.
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  - [23] ASME B89.1.6-2002, Measurement of Plain Internal Diameters for Use as Master Rings or Ring Gages.
  - [24] ASME B89.1.5-1998, Measurement of Plain External Diameters for Use as Master Discs or Cylindrical Plug Gages.
  - [25] ANSI/NCSL Z540.3-2006 (R2013), Requirements for the Calibration of Measuring and Test Equipment.
  - [26] JCGM 102:2011, Evaluation of measurement data — Supplement 2 to the "Guide to the expression of uncertainty in measurement" — Extension to any number of output quantities.
  - [27] JCGM 106:2012, Evaluation of measurement data — The role of measurement uncertainty in conformity assessment.





# NONMANDATORY APPENDIX A

## SAMPLE DIMENSIONAL MEASUREMENT PLAN

### A-1 PURPOSE

This Nonmandatory Appendix discusses the development of a dimensional measurement plan. A simple workpiece is used for illustration, but the same methods can be used for all workpieces.

Note that for more complex workpieces, which include both individual and related features, geometric characteristics can have a significant influence on measurement uncertainty (which includes the effects of both manufacturing and measurement). Certain geometric components of the product definition are crucial in achieving a satisfactory measurement process. Incomplete or erroneous understanding of these requirements may lead to a bias in the measurement results.

In order to limit the introduction of such a bias when developing the measurement process, particular attention should be given to the following:

- (a) specified datums and controls used to qualify the datum features
- (b) the interrelationship of the datums to define a datum reference frame, including functional datum precedence and the geometric controls affecting the necessary relationships
- (c) effects of form deviation and surface texture on the measurement of features of size, and other geometric controls influenced by form
- (d) effects of form, orientation, and other types of geometric variation on virtual condition measurements

Often the development of an acceptable dimensional measurement plan is not a linear process. When difficulties are encountered, returning to one or more previous steps may be necessary.

### A-2 LOCATING PIN

#### A-2.1 Background

The company producing this pin has a long history of manufacturing similar workpieces. At the time this order must be fulfilled, there is only one machine tool (a centerless grinder) available and this machine is known to produce a form error of 2  $\mu\text{m}$  on this type of workpiece. From

extensive historical data (see para. A-2.2.3), it is known that this form error is stable. The pin diameter is relatively stable over a period of an hour, but slowly and randomly drifts over a period of several hours. For example, when a histogram of the actual mating size (AMS) of the pin diameter is examined, based on high-accuracy laboratory measurements collected over a full production run (without process control), the distribution is approximately Gaussian and has a standard deviation of 1.8  $\mu\text{m}$ ; most of this spread in diameters is due to drift. Previously, this level of accuracy was sufficient for the jobs assigned to this machine, but in this example, the tight tolerances of the pin diameter require the use of in-process control to keep the diameter from drifting out of tolerance during the 278-hr production run. The DMP must develop the gaging requirements and the process control limits to meet the required specification.

#### A-2.1.1 Review and Understand Workpiece Drawing.

The workpiece used to illustrate development of a dimensional measurement plan is a steel locating pin, shown in Fig. A-2.1.1-1.

(a) The actual mating size (AMS) of the pin cylindrical surface is the smallest cylinder of perfect form such that all points on the pin cylindrical surface lie on or within the cylinder and must be no greater than 25 mm.

(b) Any actual local size (ALS) of the cylindrical surface must be no less than 24.99 mm; in accordance with ASME Y14.5.1, this means that any dimensional cross section of the cylindrical surface must admit an inscribed sphere of perfect form with a diameter of no less than 24.99 mm.

(c) The AMS of the length must be no greater than 75 mm; that is, all points on the ends of the pin must lie on or between two parallel planes no greater than 75 mm apart.

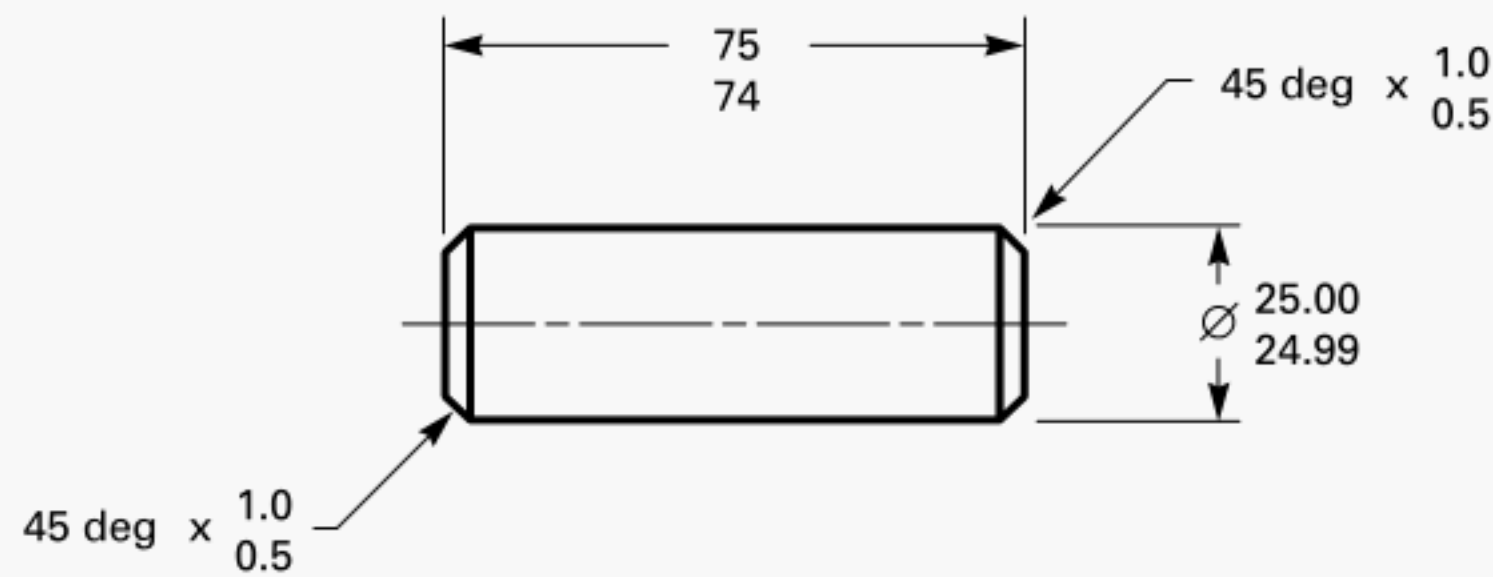
(d) The ALS of the length must be no less than 74 mm; that is, any two opposite points on the pin ends must be at least 74 mm apart.

The DMP requests clarification on the 45-deg chamfer and is told that visual inspection is adequate. This is confirmed by a memorandum.





Fig. A-2.1.1-1



GENERAL NOTE: Steel locating pin drawing per ASME Y14.5-2009 and ASME Y14.5.1-1994; all dimensions are in millimeters.

**A-2.1.2 Review Manufacturing Plan and Manufacturing Process Failure Mode and Effects Analysis.** In this step, the probable stability of the manufacturing process, probable modes and consequences of process failure, and workpiece quantities are identified from the manufacturing plan. This background information is useful for determining what measurements should be made and what sampling plan should be used.

The DMP determines the following considerations to be significant:

- (a) Total production is 50,000 pieces. Production rate is 180 pieces per hour.
- (b) Blanks are produced on a screw machine and heat treated. Experience indicates that thermal distortion is within the stock allowance for final finishing. Each piece will have cutoff projections, which must be within the length tolerance.
- (c) Final finishing is by through-feed centerless grinding. Experience indicates that this will cause lobing, but the process is stable once it is set up. Allowance must be made for lobing when planning the measurement process (see para. A-2.2.9).
- (d) The major process failure mode, determined from experience, is regulating wheel infeed error; failure to adjust the infeed causes significant workpiece diameter variation drift over a period of hours.

**A-2.1.3 Dimensional Measurement Planning Constraints.** In the approach recommended by this Standard, the most common constraints are the measurement uncertainty required to yield acceptable probabilities of pass and fail errors, and measurement cost. In some cases there will be other constraints. There is usually a tradeoff between measurement uncertainty and measurement cost. In cases of 100% inspection, with a production distribution having a significant number of nonconforming workpieces and a measurement system with a significant amount of uncertainty, increasing the probability of accepting conforming workpieces comes at a cost of also increasing the number of conforming workpieces rejected. If the cost of a workpiece is high, then this is an expensive means to improve the quality of the product. In the exam-

ple below, process control measurements with regular adjustment of the machine tool are described. This has the advantage of reducing the number of rejected workpieces (both conforming and nonconforming) and reducing the total number of measurements performed. In the example of this Nonmandatory Appendix, the cost of this method includes having available a large amount of laboratory-quality historical measurement data on similar workpieces produced on this machine. Additionally, there is the cost of performing measurements in the production facility (at the machine tool) and the cost of scrapping all the workpieces produced between process control measurements should a measurement indicate the production system was out of control.

In this example, the historical data show that the machine tool will produce a significant number of nonconforming workpieces given the tight diameter tolerance on this workpiece, but due to a slow drift of the machine tool, it is determined that frequent process control measurements can be used to readjust the machine tool, counteracting the drift and preventing nonconforming workpiece production. Alternatively, a more expensive machine tool might be used to reduce the width (standard deviation) of the pin production distribution and correspondingly increase the number of conforming pins and decrease the number of nonconforming pins, but an economic analysis determines that (for this example) the process control procedure is preferred.

In some cases, constraints will be determined by contract or provided by management. In other cases, the DMP will develop the constraints and submit them with the dimensional measurement plan for approval. In this sample plan, the DMP develops the constraints by the following reasoning:

- (a) The pin will be used to control the relative location of two workpieces.
- (b) Pin diameter is the tightest toleranced feature; the designer has established these tolerances considering the consequences if the AMS of the pin is too large it might not fit in the mating hole and if the ALS is too small, there will be a negative impact on function.



- (c) The cost of rejecting a conforming pin is small.
- (d) Economic analysis determines that the available centerless grinder is the most profitable means of production even in consideration that it is working near the limits of its capability relative to the required tolerances; for example, from prior high-accuracy laboratory measurements, the DMP knows that about 5% of the pin production run will be nonconforming to the AMS specification if process control is not used (see the methods described in para. A-2.2.10.1).
- (e) The DMP estimates that, subject to approval, 100% inspection will not meet the cost constraints for this job, but that funds will be available for a moderate amount of special gaging for process control to ensure that the requirements are met.
- (f) High-accuracy measurements have been performed on many pins produced by this machine over many prior months, hence it is known that the pin has a stable form error but that variation in the diameter of the pin is slowly varying (due to gradual wear of the grinding wheel and thermal drift in the grinder) and that it is reasonable to assume that all pins produced between two acceptable process control measurements are also acceptable.
- (g) From an economic analysis, the probability of a pass error for the AMS must be less than 0.001, i.e., 0.1%.
- (h) ALS is critical and an economic analysis determines that the probability of a pass error must be less than 0.0005 (i.e., 0.05%).

**A-2.1.4 Existing Equipment.** The DMP determines that all necessary instruments and gages for measuring the locating pins are available.

## A-2.2 Plan Development

By performing the following steps, the DMP ensures that the content of the plan follows the recommendation of para. 4.3.

**A-2.2.1 Measurements to Be Taken.** The DMP verifies that length properties of a pin are controlled in the process for producing the blank. Thus diameter is the only property to be controlled in the example dimensional measurement plan of this Nonmandatory Appendix.

Since diameter may drift during production, process control measurements will be required.

**A-2.2.2 Applicable Standards and Technical Reports.** The DMP considers the applicability of each of the following documents:

- ASME Y14.5-2009, Dimensioning and Tolerancing.
- ASME Y14.5.1M-1994, Mathematical Definition of Dimensioning and Tolerancing Principles.

- ASME Y14.43-2011, Dimensioning and Tolerancing Principles for Gages and Fixtures. The provisions of this standard are applicable to the specification of suitable gaging for verification of the actual mating size of the pins.
- ASME B89.3.1-1972, Measurement of Out-of-Roundness. The provisions of this standard are applicable. Form errors can be significant contributors to the uncertainty of size measurements on cylindrical workpieces. Such errors will be estimated based on out-of-roundness measurements taken in the quality laboratory.
- ASME B89.6.2-1973, Temperature and Humidity Environment for Dimensional Measurement.
- ISO/IEC 17025:2005, General requirements for the competence of testing and calibration laboratories. Meeting the requirements of this standard is good practice and is often required to support an assertion of metrological traceability. The DMP determines that meeting the requirements is ensured by compliance with provisions of the company gage control system.
- JCGM 100:2008, Evaluation of measurement data — Guide to the expression of uncertainty in measurement (GUM).
- ANSI/NCSL Z540.3-2006 (R2013), Requirements for the Calibration of Measuring and Test Equipment. The DMP determines that meeting the requirements is ensured by compliance with provisions of the company measuring and test instrument calibration control system.
- ASME B89.7.3.1-2001, Guidelines for Decision Rules: Considering Measurement Uncertainty in Determining Conformance to Specifications. This standard is used to communicate the decision rules selected for the acceptance/rejection measurements.
- ASME B89.7.3.2-2007, Guidelines for the Evaluation of Dimensional Measurement Uncertainty, and ISO/TR 16015:2003, Geometrical product specifications (GPS) — Systematic errors and contributions to measurement uncertainty of length measurement due to thermal influences. These technical reports contain valuable guidance applicable to the evaluation of measurement uncertainty associated with the results of locating pin measurements.
- ASME B89.7.4.1-2005, Measurement Uncertainty and Conformance Testing: Risk Analysis. The DMP considers the approach of this technical report to be appropriate for the calculation of the probabilities of pass errors and fail errors.
- ASME B89.7.5-2006, Metrological Traceability of Dimensional Measurements to the SI Unit of Length. The DMP determines that it is necessary for contractual reasons to assert metrological





traceability, per the interpretation of ASME B89.7.5, of the results of pin diameter measurements.

The foregoing is not intended as an exhaustive list of applicable standards and technical reports. It is intended only as an example of how the applicability of available documents is determined.

**A-2.2.3 Process Qualification Data.** The company has a long history of manufacturing pins similar to this on the same centerless grinder that will be used on this job. As part of the company quality policy, pins have been removed from the production line at recorded times and measured in detail for their form and diameter errors on very high-accuracy laboratory instruments. The pins measured in the laboratory are inspected with a high-density sampling strategy (number and location of measurement points) with a stylus sufficiently small to assess the effects on any relevant form error.

From these measurements, it is known that the manufacturing process is relatively stable but the pin diameter slowly and randomly drifts over a period of hours. This data will be used in the calculation of process control limits and to establish the inputs to the uncertainty analysis. The DMP determines that the manufacturing process can be kept in control by periodic process control measurements and grinder adjustments.

The manufacturing process produces a three-lobed form error that is relatively large with regard to the tolerances of this particular pin design. Nevertheless, the production method is relatively stable and is the most economical method of production.

Based on the laboratory measurements, the average peak-to-valley roundness error for the production process is 2  $\mu\text{m}$ . Expanded uncertainties for these laboratory measurements are less than 0.1  $\mu\text{m}$  and can thus be neglected in the evaluation of size uncertainty.

From the high-density, high-accuracy laboratory data (on workpieces produced without process control), the distribution of the AMS of the pins is known to be Gaussian, centered 2  $\mu\text{m}$  larger than the nominal (target) diameter due to the lobing error, and the distribution has a standard deviation of 1.8  $\mu\text{m}$ . Similarly, the distribution of the ALS on pins also appears Gaussian, centered 2  $\mu\text{m}$  smaller than the nominal (target) diameter, and the distribution also has a standard deviation of 1.8  $\mu\text{m}$ . The 1.8- $\mu\text{m}$  standard deviation is over a long production run and is due to the slow drift of the grinder and short-term random effects; this drift will result in a significant number of nonconforming pins if left uncorrected.

The relationship between the AMS and ALS diameters is fairly stable and both drift together in a correlated manner. From the high-accuracy data, the form error is known to be three-lobed with a mean 2- $\mu\text{m}$  peak-to-valley form error. The variation in the peak-to-valley form

error has a standard deviation of 0.3  $\mu\text{m}$  on a pin-to-pin basis. The distribution of the differences between the AMS and ALS diameters has a mean of 4  $\mu\text{m}$  and a standard deviation of 0.6  $\mu\text{m}$  on a pin-to-pin basis. Analysis of the laboratory measurements shows that the surface finish of the pin is negligible.

**A-2.2.4 In-Process Measurements to Be Taken.** For process control of the production workpieces, the DMP decides that one workpiece will be measured every 12 min. This decision is based on an assessment of the stability of the manufacturing process. The DMP decides to use a process control chart for monitoring the pin diameter.

The ALS of each pin is estimated based on a two-point measurement taken with a high-accuracy micrometer. Each measurement will be corrected by  $-2 \mu\text{m}$  to account for the  $+2\text{-}\mu\text{m}$  systematic error associated with using a two-point measurement procedure (see Fig. A-2.2.4-1).

Similarly, the AMS diameter can be estimated by the two-point diameter measurement and corrected by  $+2 \mu\text{m}$ . Each pin used in process measurements will also be passed through a ring gage to directly confirm that the AMS specification is met.

The DMP determines that that pins will be visually inspected for faults such as burrs and scratches. Visual inspection ensures that the entire diametrical surface is ground, and that no handling damage has occurred.

**A-2.2.5 Gage Selection.** For compliance with the AMS specification, the DMP chooses a XXX ring gage with a bore depth equal to the length of the pin and with a tolerance of  $\pm 0.19 \mu\text{m}$ . Reference to catalogs and discussions with a gage manufacturer show that this gage can be obtained at an acceptable cost.

For compliance with the ALS specification and for process control, the DMP chooses an electronic indicating high-accuracy micrometer that has the accuracy needed to meet the chosen probabilities of pass and fail errors.

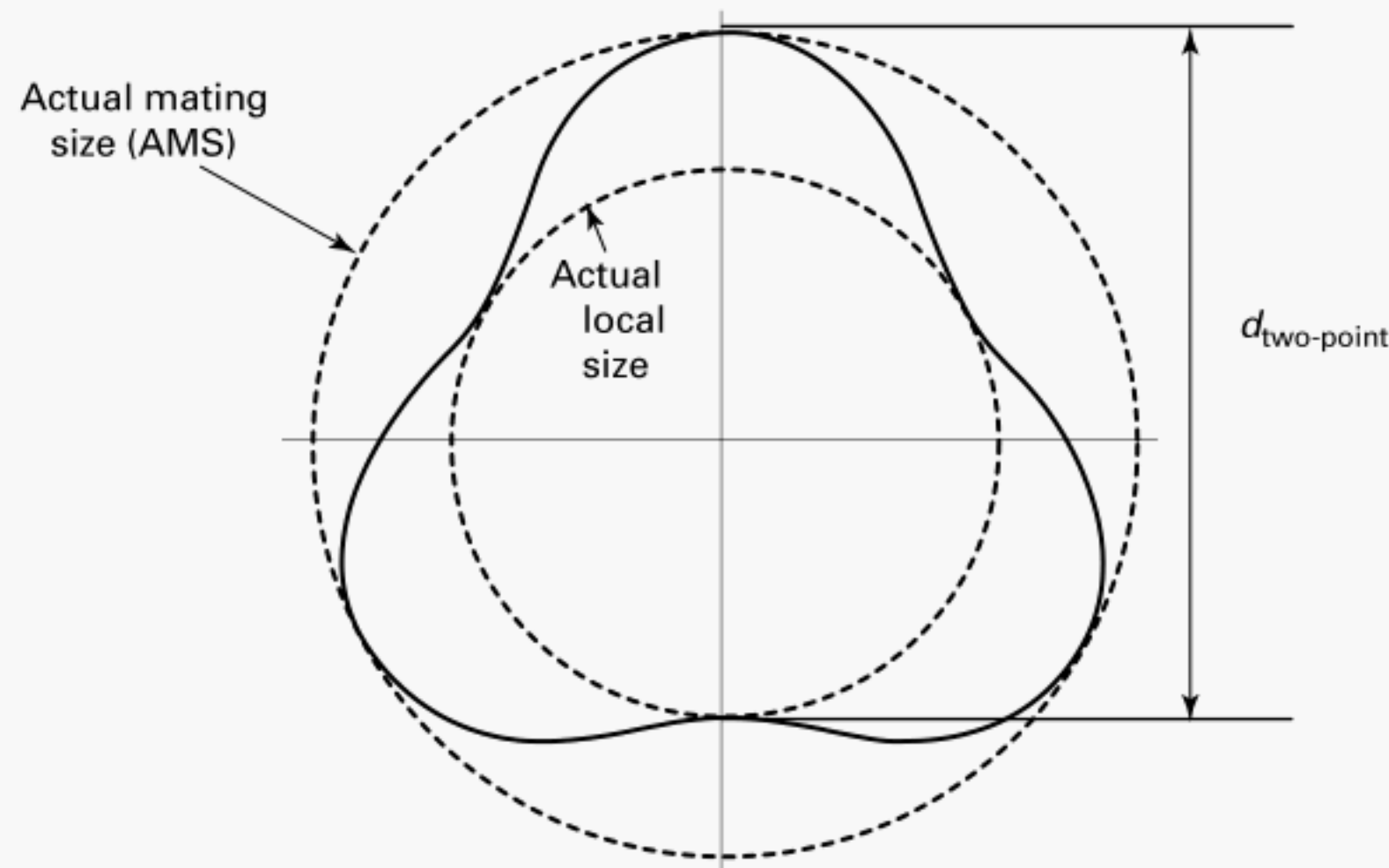
**A-2.2.6 Skill Level and Time Availability.** The DMP verifies by reference to the manufacturing plan that qualified personnel will be available to make measurements.

**A-2.2.7 Location.** The DMP determines that a station at the grinding machine will be used for the measurements. The machine operator will take and process the measurements and enter the results on the process control charts, and adjust the machine for process control. This decision is based on cost effectiveness. The DMP determines on the basis of experience that the machine operator is qualified to perform these functions. The DMP verifies that workstation-related factors, ergonomics, and environment for all measurements are determined to be adequate.





Fig. A-2.2.4-1



GENERAL NOTE: Profile of a pin displaying three-lobed roundness error. Such a workpiece is characterized by a constant two-point diameter,  $d_{\text{two-point}}$ , as measured by a high-accuracy micrometer. The two-point diameter is larger than the ALS by an amount equal to the peak-to-valley roundness error and smaller than the AMS by the same amount.

**A-2.2.8 Measurement Time.** The DMP determines that a sample is taken every 12 min and that a single operator has adequate time.

**A-2.2.9 Evaluation of Measurement Uncertainty.** It is imperative that the DMP fully understand the meaning represented by each measurand specified by the drawing. The following discussion highlights key error influences for a diameter per ASME Y14.5. The DMP must evaluate each of the uncertainty sources and determine which ones will have a direct influence on the respective measurements. The specific uncertainty sources and their magnitudes depend on the manufacturing process used to manufacture the workpieces; see Table A-2.2.9-1 for some workpiece contributors.

The DMP considers the measuring tasks in light of the list of influence quantities in Nonmandatory Appendix C of ASME B89.7.3.2 and determines those in Table A-2.2.9-2 to be applicable.

**A-2.2.9.1 Actual Mating Size (AMS) Uncertainty Using Ring Gage.** Measurement uncertainty associated with measurement of the actual mating size (AMS) is evaluated as follows:

(a) *Thermal Issues.* The DMP determines the workpieces coming off the machine with the temperature controlled by chilled coolant are within  $20^{\circ}\text{C} \pm 3^{\circ}\text{C}$ .

The pin temperature,  $T$ , during measurement is thus known to be in the interval  $17^{\circ}\text{C} \leq T \leq 23^{\circ}\text{C}$ . The DMP assumes that temperatures near the limits are unlikely, and thus assigns a triangular probability distribution to characterize knowledge of the workpiece temperature.

The standard uncertainty of the pin temperature is then given by (see ASME B89.7.3.2, section B-3)

$$u(T) = 0.41 \times \frac{(23 - 17)}{2}^{\circ}\text{C} = 1.23^{\circ}\text{C}$$

The CTE for a steel pin is taken to be  $\alpha_{\text{pin}} = 11.5 \times 10^{-6}/^{\circ}\text{C}$ . Since the best estimate of the pin temperature is  $20^{\circ}\text{C}$ , the standard uncertainty component of the pin diameter associated with uncertainty in the CTE is equal to zero. The thermally related uncertainty component of the nominal 25-mm diameter is then given by

$$\begin{aligned} u_{\text{temp pin}} &= d_{\text{nom}} \times \alpha_{\text{pin}} \times u(T) \\ &= 0.025 \times 11.5 \times 10^{-6} \times 1.23 \approx 0.35 \mu\text{m} \end{aligned}$$

The DMP determines the temperature of the ring gage is controlled within  $20^{\circ}\text{C} \pm 1^{\circ}\text{C}$ .

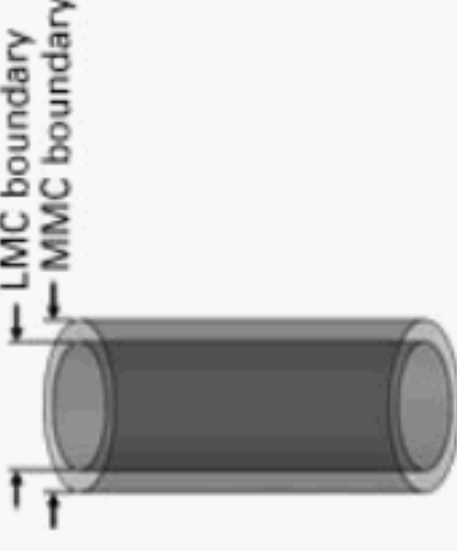
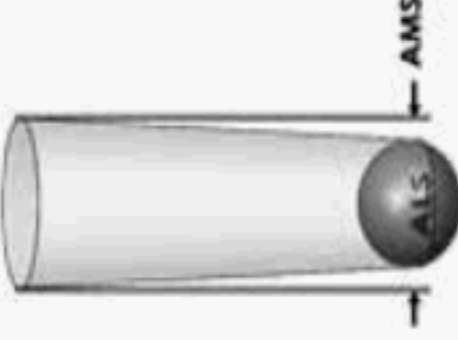
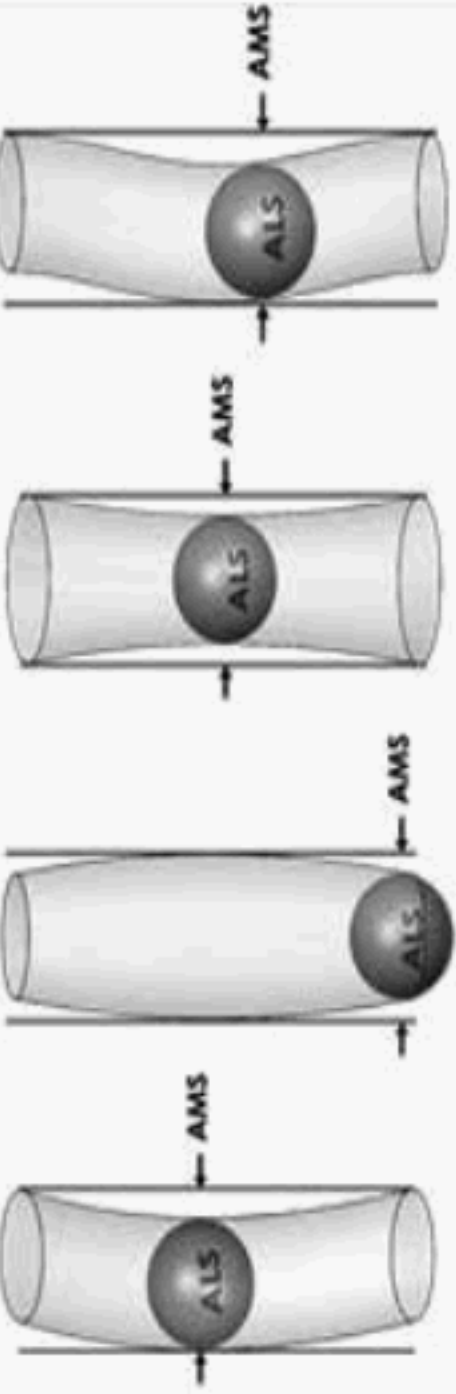
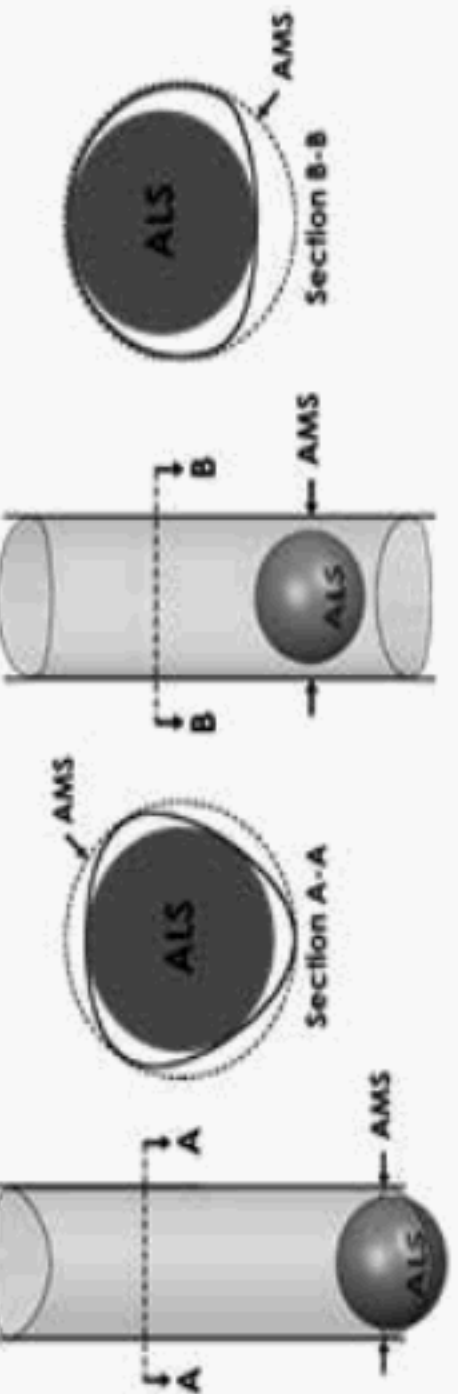

The ring gage temperature,  $T$ , during measurement is thus known to be in the interval  $19^{\circ}\text{C} \leq T \leq 21^{\circ}\text{C}$ . The DMP assumes that temperatures to be equally likely anywhere in this interval and thus assigns a uniform probability distribution to characterize knowledge of the workpiece temperature. The standard uncertainty of the gage temperature is then given by (see ASME B89.7.3.2, section B-3)

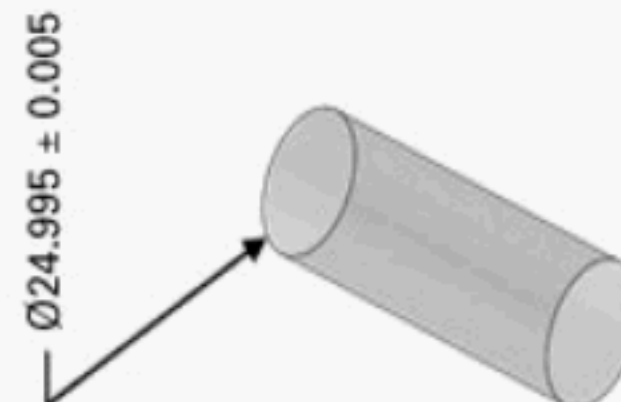
$$u(T) = 0.58 \times \frac{(21 - 19)}{2}^{\circ}\text{C} = 0.58^{\circ}\text{C}$$

The CTE for the ring gage is taken to be  $\alpha_{\text{gage}} = 11.5 \times 10^{-6}/^{\circ}\text{C}$ . Since the best estimate of the pin temperature is  $20^{\circ}\text{C}$ , the standard uncertainty component of the gage diameter associated with uncertainty in the CTE



Table A-2.2.9-1 Geometrical Characteristics and Associated Form Errors

Geometric Requirement	Direct Contributors	Comments	Supporting Graphics
	Range in size	Size cannot exceed the boundary of perfect form at its maximum material condition (MMC), which in this case is 25.000, or a two-dimensional boundary of perfect form at its least material condition (LMC), which in this case is 24.990. Note: There is no requirement for these boundaries to be concentric.	
	Taper	Size allows the pin to have taper as long as the three-dimensional actual mating size (AMS), which would be the smallest circumscribed cylinder, is not larger than the MMC, and the two-dimensional actual local size (ALS), which is the maximum inscribed sphere at any local cross section, is not smaller than the LMC.	
	Straightness	Size allows the pin to be out of straight a maximum amount equal to the total size tolerance (0.010) as long as the AMS does not violate the MMC and the ALS does not violate the LMC.	
	Roundness	Size allows the pin to be out of round a maximum amount equal to the total size tolerance (0.010) as long as the AMS does not violate the MMC and the ALS does not violate the LMC.	
	Surface finish	Surface roughness influences size.	





**Table A-2.2.9-2 Influence Quantities Used in This Example**

	Actual Mating Size (Ring Gage)	Micrometer Measurement (Two-Point)
Temperature [Note (1)]	x	x
Thermal drift		x
Form		x
Form variation		x
Gage tolerance	x	x
Operator effects/reproducibility		x

**NOTE:**

(1) The uncertainty associated with the coefficients of thermal expansion are negligible because the gage and pin have the same CTE and the mean temperature is 20°C.

is negligible. The thermally related uncertainty component of the nominal 25-mm diameter is then given by

$$u_{\text{temp gage}} = d_{\text{nom}} \times \alpha_{\text{gage}} \times u(T) \\ = 0.025 \times 11.5 \times 10^{-6} \times 0.58 \approx 0.17 \mu\text{m}$$

It is further assumed that the temperature of the ring gage (determined by the air temperature of the room) and the temperature of the pin (determined by the chilled coolant) are uncorrelated.

(b) *Ring Gage.* A XXX ring gage is ordered to a nominal size of 24.9995 mm with a tolerance of  $\pm 0.19 \mu\text{m}$ ; see para. A-2.2.10.1 for the calculations associated with the nominal size of the ring gage. See ASME B89.1.6-2002 for different classes of gages and their associated tolerances.

Since the sampling plan requires a total of 1,389 pins to be measured, wear is not considered a significant factor.

Since the gagemaker's calibration uncertainty is unknown, the DMP assumes that the  $\pm 0.19 \mu\text{m}$  tolerance is a uniform distribution centered at 24.9995 mm with a half-width of  $0.19 \mu\text{m}$  to model the ring gage. The associated standard uncertainty component is

$$u_{\text{gage}} = \frac{0.19}{\sqrt{3}} \mu\text{m} \approx 0.11 \mu\text{m}$$

The DMP considers all the influence quantities to be uncorrelated. The combined standard uncertainty associated with measurement of the actual mating size (AMS) is then

$$u_{\text{AMS ring gage}} = \sqrt{u_{\text{temp pin}}^2 + u_{\text{temp gage}}^2 + u_{\text{gage}}^2} \\ = \sqrt{(0.35)^2 + (0.17)^2 + (0.11)^2} = 0.40 \mu\text{m}$$

With  $k = 2$  expanded uncertainty,  $U_{\text{AMS ring gage}} = 0.80 \mu\text{m}$ .

**A-2.2.9.2 Actual Local Size (ALS) Uncertainty Using a High-Accuracy Micrometer.** The uncertainty

associated with measurement of actual local size (ALS) is evaluated as follows:

(a) *Form.* A three-lobed form error will yield the same measurement value for a two-point diameter measurement regardless of the rotational orientation of the pin. For a pure three-lobed cross section, a measured two-point diameter will overestimate the diameter of the ALS by an amount equal to the peak-to-valley roundness error (see Fig. A-2.2.4-1). The average three-lobed form error on the pin is  $2 \mu\text{m}$ , thus a correction is applied to the measured two-point diameter measurement to estimate the ALS measurand.

If the pin only had a stable pure three-lobed cross-sectional form error then the  $2\text{-}\mu\text{m}$  correction would fully account for this effect and there would be no issue with the sampling strategy (number and location of the two-point measurements). However, there is a variation to the three-lobed form error, which has a standard deviation of  $0.3 \mu\text{m}$ , and this is an uncertainty source associated with the correction.

$$u_{\text{form}} = 0.30 \mu\text{m}$$

(b) *Temperature.* The considerations for the production pin measured with the high-accuracy micrometer are similar to the case for the AMS measurement. The production pin has a temperature control of  $20^\circ\text{C} \pm 3^\circ\text{C}$  resulting in a standard uncertainty of its diameter of  $0.35 \mu\text{m}$ . The high-accuracy micrometer is zeroed (mastered) on a high-accuracy calibrated gage pin. While the high-accuracy micrometer and calibration pin can be assumed to be at the same temperature (they are placed close by each other in the room), the uncertainty associated with the  $20^\circ\text{C} \pm 1^\circ\text{C}$  temperature of the calibration pin must be included; this calculation is similar to the AMS case and yields a standard uncertainty of  $0.17 \mu\text{m}$ . Since both the gage pin and production pin have the same nominal CTE and their mean temperature is  $20^\circ\text{C}$ , the effects due to the uncertainties in the CTE values are negligible.

The high-accuracy micrometer is mastered against the calibration pin once an hour; the DMP estimates that



the temperature of the high-accuracy micrometer could drift by 0.7°C during this time, resulting in another standard uncertainty of 0.12  $\mu\text{m}$ . The standard uncertainty of these temperature effects is then

$$u_{\text{temp}} = \sqrt{(0.35)^2 + (0.17)^2 + (0.12)^2} = 0.41 \mu\text{m}$$

(c) *High-Accuracy Micrometer Calibration.* A gage pin is ordered with a nominal size of 24.995 mm with a tolerance of  $\pm 0.15 \mu\text{m}$  and used to set the high-accuracy micrometer; see ASME B89.1.5-1998.

An additional calibration effect associated with the operator during the mastering process includes any inaccuracies in the micrometer scale and reproducibility effects; based on experience, it is estimated to be within  $\pm 0.3 \mu\text{m}$ .

Given this information, the DMP assigns uniform distributions to these two errors, yielding associated standard uncertainty components of 0.09  $\mu\text{m}$  and 0.17  $\mu\text{m}$ , respectively. The resulting gage calibration standard uncertainty is then

$$u_{\text{cal}} = \sqrt{(0.09)^2 + (0.17)^2} = 0.2 \mu\text{m}$$

(d) *Gage Resolution.* An indicator with a resolution of 0.05  $\mu\text{m}$  is chosen for the high-accuracy micrometer. The associated uncertainty component is taken to be negligible.

(e) *Pin Measurement Operator Effects.* In addition to setting the high-accuracy micrometer on the calibration pin, the operator also must measure the production pin and has a similar reproducibility problem leading to a standard uncertainty of  $u_{\text{oper}} = 0.17 \mu\text{m}$ .

Consideration of the contributing influence quantities indicates that there are no significant correlations. The combined standard uncertainty associated with measurement of the diameter of ALS is then

$$\begin{aligned} u_{\text{ALS two-point}} &= \sqrt{u_{\text{form}}^2 + u_{\text{temp}}^2 + u_{\text{cal}}^2 + u_{\text{oper}}^2} \\ &= \sqrt{(0.30)^2 + (0.41)^2 + (0.20)^2 + (0.17)^2} \\ &= 0.57 \mu\text{m} \end{aligned}$$

With  $k = 2$  expanded uncertainty,  $U_{\text{ALS two-point}} = 1.14 \mu\text{m}$ .

**A-2.2.10 Acceptance Limits.** Only a small fraction of the production pins will be measured. Based on historical data it is known that the grinder is slowly varying over a period of hours affecting the pin diameter. To control this effect a pin will be measured using both the high-accuracy micrometer and the ring gage every 12 min. If two successive pin measurements are acceptable then all pins produced between those two measurements are considered acceptable. In the following risk calculations, the function  $f(x, x_m)$  is the joint probability density function (PDF) of the possible true values  $X$  of the dimension, and possible measured values  $X_m$  produced by the measuring system; details

regarding the acceptance limit calculations can be found in Nonmandatory Appendix D. The production distribution will be considered as the ordinary (nonprocess-controlled) distribution for the grinder; process control would further narrow the distribution and hence further lower the associated risk analysis limits.

#### A-2.2.10.1 AMS Acceptance Limit Using a Ring Gage

(a) The process distribution without process control is a Gaussian PDF [see eq. (D-3)], with mean value,  $x_0 = 24.997 \text{ mm}$  and standard deviation,  $u_p = 0.0018 \text{ mm}$ .

Note that the production process without process control will produce approximately 5% nonconforming pins; the goal of the process control measurements is to reduce this number to within the risk analysis requirements

Nonconforming based on the AMS specification

$$= \int_{25}^{\infty} f(x) dx \approx 4.7\%$$

(b) The measuring system (ring gage) distribution is a Gaussian PDF [see eq. (D-4)], with standard uncertainty  $u_m = u_{\text{AMS ring gage}} = 0.40 \mu\text{m}$ .

(c) The AMS tolerance limits are given by the pin specification. The gaging limits are determined by computing the risk integrals and adjusting the gaging limits so that the risk analysis requirements are fulfilled.

$$\begin{aligned} G_L &= 0 \text{ mm} \\ G_U &= 24.9995 \text{ mm} \\ T_L &= 0 \text{ mm} \\ T_U &= 25.0000 \text{ mm} \end{aligned}$$

Hence, the requirement that the probability of a pass error  $\leq 0.001$  can be achieved using an XXX ring gage with a 24.9995-mm diameter; this corresponds to a 63%  $U_{\text{AMS ring gage}}$  stringent acceptance decision rule as confirmed by the calculation shown below.

(d) Upper limit pass error based on the AMS specification ( $G_L \leq X_m \leq G_U$  and  $X > T_U$ )

$$R_c(\text{upper}) = \int_0^{24.9995} \int_{25}^{\infty} f(x, x_m) dx dx_m \approx 0.1\%$$

#### A-2.2.10.2 ALS Acceptance Limits Using a High-Accuracy Micrometer

(a) The process distribution without process control is a Gaussian PDF [see eq. (D-3)], with mean value,  $x_0 = 24.9930 \text{ mm}$  and standard deviation,  $u_p = 0.0018 \text{ mm}$ .

(b) The measuring system (high-accuracy micrometer) distribution is a Gaussian PDF [see eq. (D-4)], with standard uncertainty,  $u_m = u_{\text{ALS two-point}} = 0.57 \mu\text{m}$ .

Note that without process control, the production process will produce a significant number of nonconforming pins





Nonconforming to the ALS specification =

$$\int_0^{24.9900} f(x)dx \approx 7.4\%$$

(c) The ALS tolerance limits are given by the pin specification. The gaging limits are determined by computing the risk integrals below and adjusting the limits so that the risk analysis requirements are fulfilled.

$$\begin{aligned} G_L &= 24.9910 \text{ mm} \\ G_U &= \infty \text{ mm} \\ T_L &= 24.9900 \text{ mm} \\ T_U &= \infty \text{ mm} \end{aligned}$$

(d) The best estimate of the ALS using a high-accuracy micrometer is the reading minus 2.0  $\mu\text{m}$ , with a  $U_{\text{ALS two-point}} = 1.14 \mu\text{m}$ . The pin is considered acceptable if the best estimate of the ALS is equal to or greater than 24.9910 mm; this is a 9%  $U_{\text{ALS two-point}}$  stringent acceptance decision rule; the requirement that the probability of an ALS pass error  $\leq 0.0005$  is confirmed by the risk calculation shown below. Lower limit pass error based on the ALS specification ( $G_L \leq X_m \leq G_U$  and  $X < T_L$ )

$$R_c(\text{lower}) = \int_{24.991}^{\infty} \int_{-\infty}^{24.9900} f(x, x_m) dx dx_m \approx 0.04\%$$

If the DMP does not use the risk analysis tools that consider the production distribution, the DMP will need to directly consider a decision rule associated with the measurement uncertainties of the AMS and ALS. To achieve a comparable level of risk without knowledge of the production distribution, larger guard bands may be necessary than those described in this example.

#### A-2.2.11 Process Distribution and Control Limits.

The high-accuracy micrometer measurements will also be used to control the grinder infeed adjustment. After each measurement, the operator will make an adjustment (using the most recent measurement value) to keep the grinder on the target diameter. If a measurement result is out of control (either above or below control limits), the operator will immediately alert the supervisor for further instructions.

The lower control limit for the two-point micrometer measurement will be used to control the ALS diameter. The upper control limit on the two-point micrometer measurements will be used to control the AMS diameter. Both control limits are set to assure the risk analysis is satisfied.

Additionally, a pin will be removed once per day and sent to the metrology laboratory for high-accuracy measurements to ensure that the form of the pin is stable in shape (three-lobed) and amplitude.

The ring gage cannot be used for process control, as it is a binary (go/no-go) gage and does not produce an

indicated value. The ring gage is used to confirm compliance with the AMS specification.

The high-accuracy micrometer is used for acceptance of the pin to the ALS specification and for process control for both the ALS and AMS.

**A-2.2.11.1 Lower Process Control Limit.** The lower control limit using the high-accuracy micrometer measurements is used to assure that the production pins will meet the ALS specification. For convenience, the control chart is plotted directly in the high-accuracy micrometer display reading values (this avoids the complications of correcting for the +2.0- $\mu\text{m}$  bias associated with the reporting ALS value), hence the lower control limit for the micrometer reading is 24.9930 mm; the assurance that the risk probabilities are met follow directly from the ALS acceptance limit case. If a micrometer measurement value is obtained below this value the process is out of control and the pin is not acceptable, production is stopped, and the supervisor is immediately alerted.

**A-2.2.11.2 Upper Process Control Limit.** To establish the upper control limit, an uncertainty evaluation of estimating the AMS using the high-accuracy micrometer is needed. Similar to the ALS case, the two-point micrometer measurement does not directly measure the measurand because it fails to detect the three-lobed form error and will underestimate the diameter by the peak-to-valley form error amount. Hence a 2- $\mu\text{m}$  correction will need to be added to the micrometer measurement result to estimate the AMS.

The uncertainty evaluation of the estimate of the production pin AMS using the high-accuracy micrometer proceeds as follows:

(a) *Form:*  $u_{\text{form}} = 0.30 \mu\text{m}$  (same evaluation as the ALS case)

(b) *Temperature:*  $u_{\text{temp}} = 0.41 \mu\text{m}$  (same evaluation as in the ALS case)

(c) *High-Accuracy Micrometer Calibration:*  $u_{\text{cal}} = 0.2 \mu\text{m}$  (same evaluation as in the ALS case)

(d) *Pin Measurement Operator Effects:*  $u_{\text{oper}} = 0.17 \mu\text{m}$  (same evaluation as in the ALS case)

$$\begin{aligned} u_{\text{AMS two-point}} &= \sqrt{u_{\text{form}}^2 + u_{\text{temp}}^2 + u_{\text{cal}}^2 + u_{\text{oper}}^2} \\ &= \sqrt{(0.30)^2 + (0.41)^2 + (0.20)^2 + (0.17)^2} \\ &= 0.57 \mu\text{m} \end{aligned}$$

With  $k = 2$  expanded uncertainty,  $U_{\text{AMS two-point}} = 1.14 \mu\text{m}$ .

#### A-2.2.11.3 AMS Control Limit Using a High-Accuracy Micrometer

(a) The process distribution without process control is a Gaussian PDF [see eq. (D-3)], with mean value,  $x_0 = 24.997 \text{ mm}$  and standard deviation,  $u_p = 0.0018 \text{ mm}$ .





(b) The measuring system (high-accuracy micrometer) distribution is a Gaussian PDF [see eq. (D-4)], with standard uncertainty,  $u_m = u_{\text{AMS two-point}} = 0.57 \mu\text{m}$ .

(c) The AMS tolerance limits are given by the pin specification. The gaging limits are determined by computing the integrals below and adjusting the limits so that the probability of a pass error  $\leq 0.001$  is fulfilled.

$$\begin{aligned} G_L &= 0 \text{ mm} \\ G_U &= 24.9992 \text{ mm} \\ T_L &= 0 \text{ mm} \\ T_U &= 25.0000 \text{ mm} \end{aligned}$$

To confirm the risk analysis, the risk calculations are shown below.

(d) Upper limit pass error based on the AMS specification ( $G_L \leq X_m \leq G_U$  and  $X > T_U$ )

$$R_c(\text{upper}) = \int_0^{24.9992} \int_{25}^{\infty} f(x, x_m) dx dx_m \approx 0.09\%$$

Using the method of Nonmandatory Appendix D, the DMP determines that an upper gaging limit of 24.9992 mm for the AMS will meet the risk analysis requirements (less than 0.1% consumer's risk). For convenience, the control chart is plotted directly in the high-accuracy micrometer display reading values (this avoids the complications of correcting for the  $-2\text{-}\mu\text{m}$  bias associated with reporting the AMS), hence the upper control limit for the micrometer reading is 24.9972 mm. If a micrometer measurement value is obtained above this threshold and the pin fails to pass through the ring gage, the process is out of control and the pin is not acceptable; production is then stopped, and the supervisor is immediately alerted. If a micrometer measurement value is obtained above this threshold and the pin passes through the ring gage, the pin is acceptable but the process is out of control and the supervisor is immediately alerted.

**A-2.2.12 Review of Plan Suitability.** The DMP determines the plan to be practical and applicable.

**A-2.2.13 Dimensional Measurement Plan Documentation.** The DMP reviews the minimum requirements defined in para. 4.3 and determined this measurement plan will include the following:

- (a) *Plan Number:* XXXX.
- (b) *Dimensional Measurement Planner:* Name.
- (c) *Workpiece and Revision Numbers:* YYYY.YY.
- (d) *Workpiece Name:* Locating pin.
- (e) *Properties to Be Measured:* Diameter (actual mating size and actual local size).
- (f) *Sampling Plan*
  - (1) use historical laboratory measurements to characterize production process
  - (2) use process control: 1 piece every 12 min
- (g) *Output Medium:* Paper report.

(h) *Plant Location:* Process control at grinder; process qualification and acceptance at inspection department location ZZZZ.

(i) *Measuring Equipment and Gages:* Ring gage S/N #####, 24.9995 mm  $\pm 0.19 \mu\text{m}$  diameter by 75 mm long for AMS acceptance; high-accuracy micrometer and master pin gage S/N ##### (24.995 mm  $\pm 0.15 \mu\text{m}$ ) for ALS acceptance and control and AMS control.

(j) *Demonstration of Metrological Traceability Per ASME B89.7.5:* Satisfy the following requirements for measurements of locating pin diameter:

- (1) clear definition of the measurand; AMS and ALS per ASME Y14.5/Y14.5.1
  - (2) identification of gages and equipment used in the measurement; see (i)
  - (3) statement of expanded measurement uncertainty consistent with the GUM; see para. A-2.2
  - (4) valid uncertainty budget; see para. A-2.2
  - (5) calibration report from an ISO/IEC 17025 [10] accredited laboratory for length standards used for high-accuracy micrometer mastering and roundness instrument calibration, accredited lab calibration reports accompanying all master gages, certificates on file for ring gage and master pin and micrometer calibration
  - (6) statement of company quality assurance policy regarding gage calibration, operator training, and environmental control: gages recalibrated yearly or upon damage; process reviewed semiannually for stability
- (k) *Data Evaluation Requirements:* Each workpiece must pass through the ring gage to pass the AMS requirement. Each measurement with the high-accuracy micrometer must not register a reading smaller than 24.9930 mm for ALS acceptance and control. The high-accuracy micrometer must not register a reading larger than 24.9972 mm for AMS control. The operator will observe the trend and adjust the machine to the target diameter after each measurement. If any workpiece exceeds the limit, call the supervisor who will determine actions required to ensure acceptable product.

(l) *Measurement Uncertainties:* For coverage factor  $k = 2$  (95% level of confidence), the relevant expanded uncertainties are (see para. A-2.2)

- (1) ring gage measurement of actual mating size:  $U_{\text{AMS ring gage}} = 0.80 \mu\text{m}$
- (2) high-accuracy micrometer measurement of actual local size:  $U_{\text{ALS two-point}} = 1.14 \mu\text{m}$
- (3) high-accuracy micrometer measurement of actual mating size:  $U_{\text{AMS two-point}} = 1.14 \mu\text{m}$

(m) *Probabilities of Pass and Fail Errors*

- (1) AMS upper limit pass:  $\leq 0.1\%$
- (2) ALS lower limit pass:  $\leq 0.05\%$

(n) *Special Instructions:* Coolant to keep workpieces within  $20^\circ\text{C} \pm 3^\circ\text{C}$ ; room controlled to  $20^\circ\text{C} \pm 1^\circ\text{C}$ .

(o) *Disposition of Workpieces:* Combine all acceptable measured workpieces with all unmeasured workpieces obtained between two acceptable process control





measurements and forward to production facility. All rejected workpieces are to be submitted for disposition; production is halted when a rejected workpiece is measured and any production pins produced after the last acceptable workpiece are rejected.

(p) *Disposition of Report:* File report in file XXXX by lot number. Forward copies to YY and ZZ.

The DMP submits the plan for approval as required by company policy.

The DMP files a copy of the plan and a record of its development, including evaluation of uncertainties, probabilities of pass and fail errors, gaging limits, and other information deemed to be useful for modifying the plan for other applications.

NOTE: From a technical and business perspective, it is important to understand that once an inspection plan and associated uncertainty analysis have been completed for a particular measurand, then they will not have to be newly created for similar measurands. They will only need to be suitably modified to account for changes in the relative importance and numerical values of the relevant input quantities, which will take considerably less time.

### A-3 POTENTIAL PROBLEMS WITH THE LOCATING PIN MEASUREMENT PLAN

#### A-3.1 Understanding the Drawing

In the example of section A-2, the DMP will refer to the appropriate revision of the relevant dimensioning and tolerancing standard (e.g., ASME Y14.5 or other) to determine the exact meaning of each specification on the drawing, and obtain help as required in understanding that standard.

If the drawing appears to be faulty (e.g., incomplete, inconsistent, or not in agreement with perceived requirements), the DMP will contact the designer to obtain clarification. If the drawing is faulty, corrections will be documented, e.g., by a revised drawing, memo, or record of conversation. The documentation will be kept with the record of plan development.

Planning will always be based on a clear and unambiguous specification of requirements, and never on guesses of design intent or user application.

**A-3.1.1 Understanding Process Control Constraints.** The DMP will work with the manufacturing engineer to establish the measurement requirements and constraints in support of the manufacturing process. The process control limits must be discussed with the manufacturing engineer for consideration to determine the level of manufacturing constraints required to maintain the manufacturing process.

#### A-3.2 Dimensional Measurement Planning Constraints

As discussed in para. A-2.1.3, the DMP determined from assembly considerations and subject to plan

approval, that acceptance of oversize pins at a 0.1% level was permissible. If, however, the pins are to be used as spare workpieces, acceptance of occasional oversize pins might not be permissible, and the DMP would set the probability of a pass error for the ring gage measurement at very close to zero. The same criteria would be applied to the ALS for pins at risk of being too small locally, which could impact function. The decision of risk is a business decision and should be discussed at or before the formal design review, as the mechanical designer would have had to take such risks into consideration in their tolerance stack up analysis. This decision is probably not made on a workpiece-by-workpiece basis but more as a broader business level practice.

#### A-3.3 Gage Selection

The DMP might conclude that a special long ring gage is too expensive, and that a standard short gage should be used. Depending on process characteristics some additional measurements of straightness might be required.

#### A-3.4 Location

If the environment for any measurement is unsatisfactory, the options are to improve the environment and/or to accept a larger uncertainty.

#### A-3.5 Skill Level and Time Available

If the sampling interval were shorter, the grinding machine operator might not have time to perform process control and plot the process control measurements. It might then be necessary to assign an additional person to do this work, or to automate the process.

#### A-3.6 Uncorrected Systematic Error

An instrument may be encountered that has a known systematic error (bias); that is, it reads incorrectly by a known amount. It is recommended, and the GUM assumes, that measurement results be corrected for known systematic errors if such errors are significant compared with the expanded uncertainty.

If, for economic or other reasons, it is decided not to make such corrections, approaches to combining known bias with uncertainty components are discussed in reference [11]. Such a decision is acceptable for accept/reject measurements, but not for calibrations.

#### A-3.7 Probabilities of Pass and Fail Errors

If requirements for probabilities of pass and fail errors (i.e., consumer's and producer's risks) cannot be met, then either the requirements must be rethought or the realized probabilities must be improved via changes in the process and/or the measuring system.



If one of the probabilities (pass error or fail error) is greater than the requirement and the other one smaller, the remedy may be to adjust the gaging limits. Moving the limits closer together will decrease the probability of a pass error and increase the probability of a fail error. Moving them apart will have the opposite effect.

If the probabilities of pass and fail errors are both too high, they can both be reduced by reducing the measurement uncertainty via improved gaging, or by modifying the manufacturing process distribution to reduce workpiece variability.

- (a) Measurement uncertainty can be reduced by
  - (1) using more accurate measuring instruments
  - (2) improving the environment
  - (3) improving the skill level of operators
  - (4) reducing process variability specifically related to workpiece form, waviness and roughness

The best approach is identified by considering the most significant influence quantities that contribute to the calculated uncertainty, constrained by issues of cost.

- (b) The process distribution can be changed by
  - (1) using a different manufacturing process
  - (2) adjusting the process more frequently
  - (3) changing machine settings to better center the process (i.e., reduce drift) with respect to the tolerance limits

### A-3.8 Review of Plan Suitability

If the plan is unsuitable in any respect, the source of the unsuitability must be identified. It is then necessary to go back to the step where the source was determined, revise that step, and then revise the subsequent steps.



## NONMANDATORY APPENDIX B

### GAGE SELECTION

#### B-1 PURPOSE

Candidate instruments and gages for a measuring task are chosen by comparing requirements of the task with capabilities of available instrumentation. The purpose of this Nonmandatory Appendix is to clarify such comparisons. The approach to comparison is illustrated by means of a simple example.

#### B-2 MEASURING TASK REQUIREMENTS

##### B-2.1 Interpretation of Drawing: Design Intent

It is imperative that the DMP fully understand the meaning represented by each dimension of a drawing. This imperative is discussed below in terms of a specific dimension and specific gages.

Consider the simple workpiece shown in Fig. B-2.1-1. Suppose the drawing on which the dimension of the figure appears complies with the requirements of ASME Y14.5-2009.

Reference to ASME Y14.5 shows that within the uncertainties discussed in Nonmandatory Appendix C, the diameter of a workpiece is within specifications if

(a) the workpiece can pass through a ring gage having a diameter of 20.1 mm and a length at least as long as the workpiece

(b) the actual local size of the cylinder, as defined by the diameter of the largest inscribed sphere of perfect form at any point along the axes of the cylinder, is no less than 20 mm

For clarification, see Fig. 2-6 of ASME Y14.5-2009 and para. 2.3 and Fig. 2-2 of ASME Y14.5.1M-1994.

##### B-2.2 Common Practices

The capabilities of a measuring instrument often do not correspond exactly to the definitions of drawing specifications and hence do not directly measure the measurand.

Referring to Fig. B-2.1-1, some common practices are

(a) make all diameter measurements with a micrometer

(b) check compliance with the maximum diameter specification (20.1 mm) with a ring gage shorter than the workpiece

(c) check compliance with the minimum diameter specification (20 mm) with a no-go ring gage

These practices may be usable if the shape and probable magnitudes of the form errors are known from experience or from the nature of the manufacturing process, and if the uncertainties that include the effects of these form errors are acceptable.

##### B-2.3 Form Errors

Possible form errors for the workpiece of Fig. B-2.1-1 are noncircular cross section, nonstraightness, and non-uniform diameter along the length of the workpiece. Such form errors are generally illustrated in Fig. B-2.3-1.

Figure B-2.3-1 schematically illustrates four geometric features of the locating pin

(a) "1" indicates the diameter of the largest inscribed sphere, which relates directly to the actual local size (ALS). The smallest value must be no less than 20 mm for the workpiece in Fig. B-2.1-1.

(b) "2" indicates a two-point measurement, made, for example, with a micrometer.

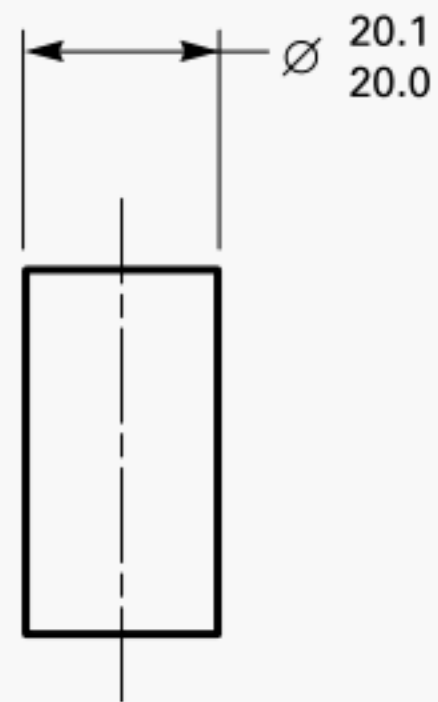
(c) "3" indicates the diameter of the smallest circumscribed circle for a particular cross section.

(d) "4" indicates the diameter of the smallest perfect cylinder that encloses all points on the workpiece surface. This relates directly to the actual mating size (AMS), which must be no greater than 20.1 mm for the workpiece in Fig. B-2.1-1.

Suppose that the DMP chooses a micrometer as the only gage to be used for diameter measurements. Clearly, additional information, e.g., form error, is needed before assessing conformity to either the AMS or ALS specifications, even if measurements are performed at all possible locations (an infinite number of measurements). Thus an uncertainty component is included in the data analysis that depends on the number and locations of the two-point diameter measurements. This uncertainty component can be evaluated through knowledge of the probable form errors.

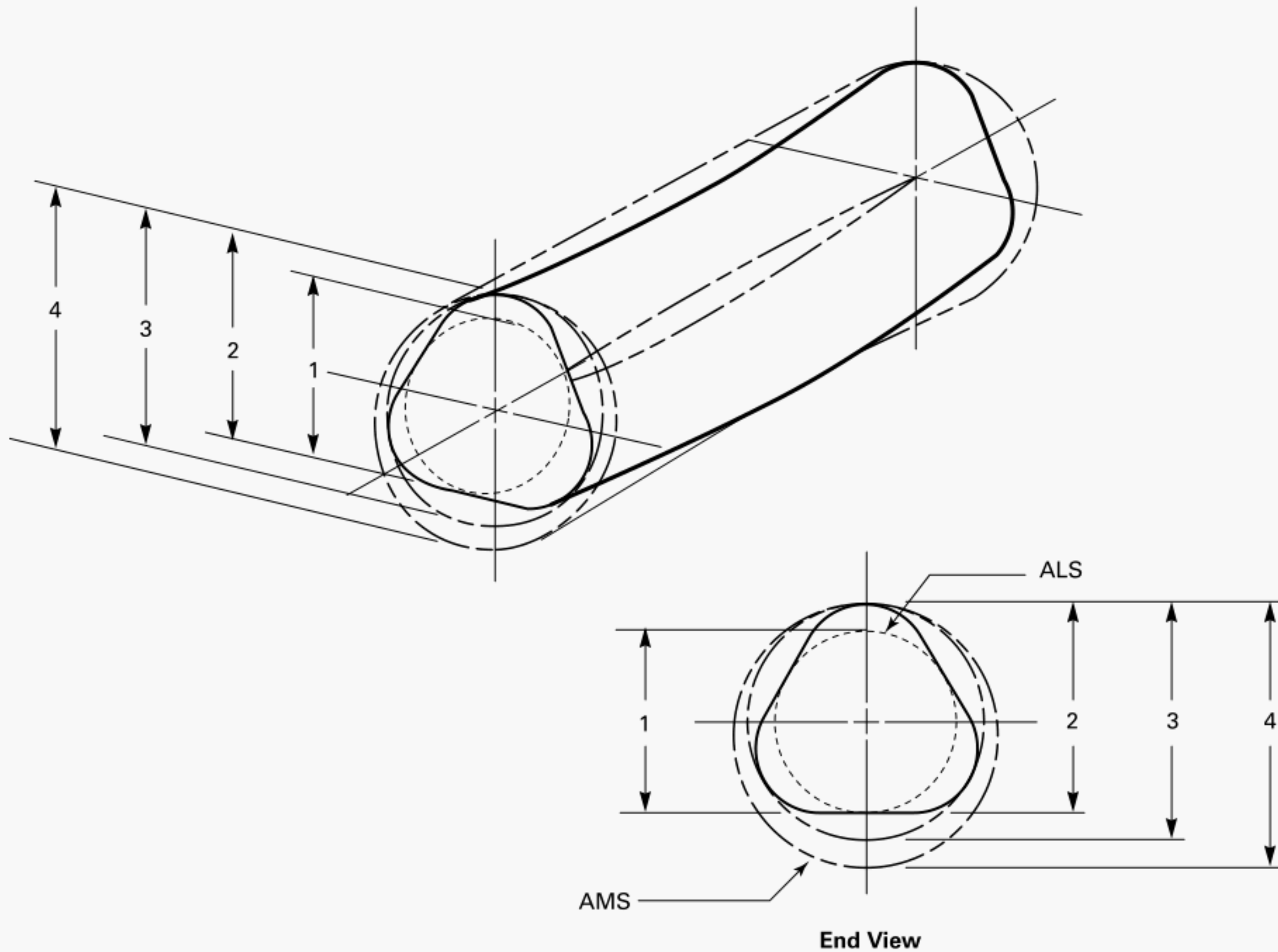


**Fig. B-2.1-1**



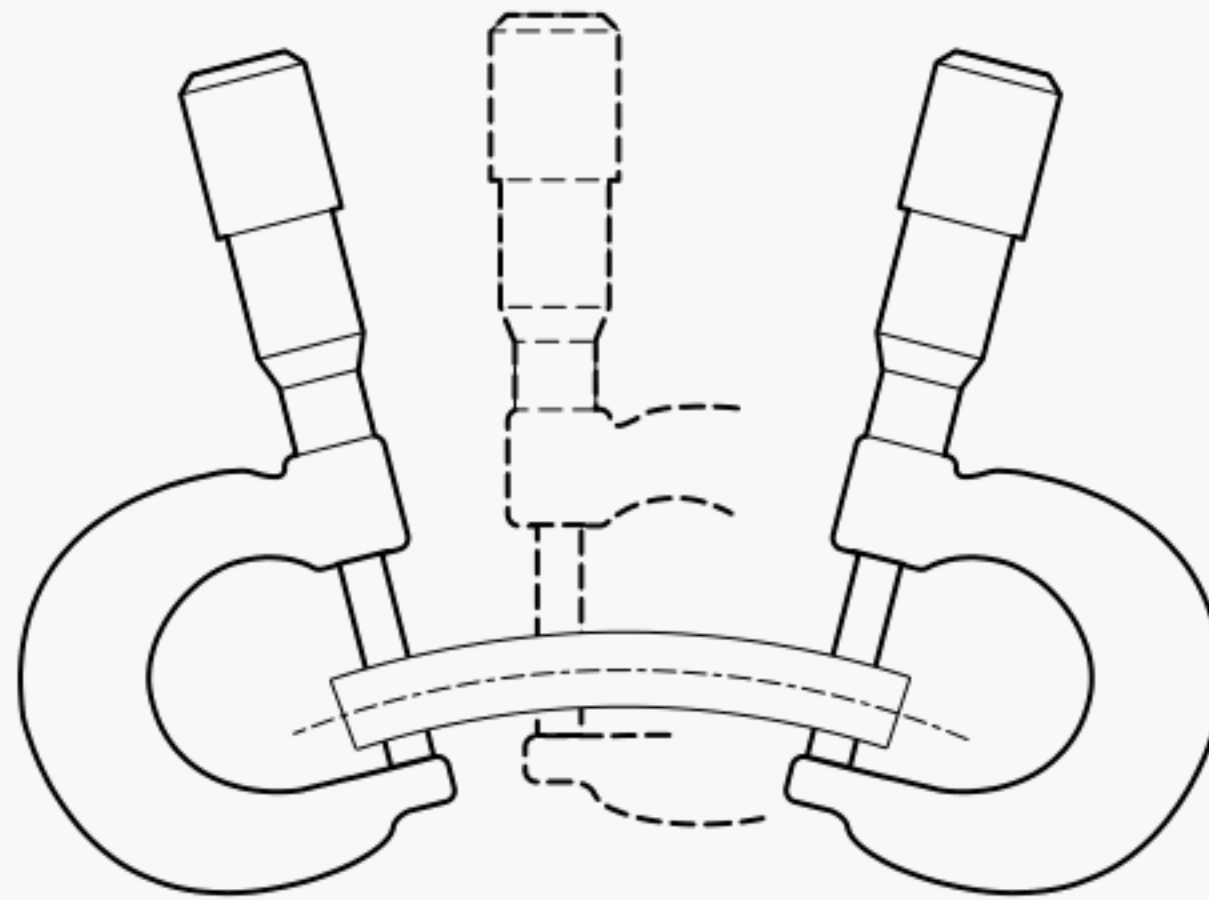
GENERAL NOTE: Drawing per ASME Y14.5-2009 and ASME Y14.5.1-1994; all dimensions are in millimeters.

**Fig. B-2.3-1**

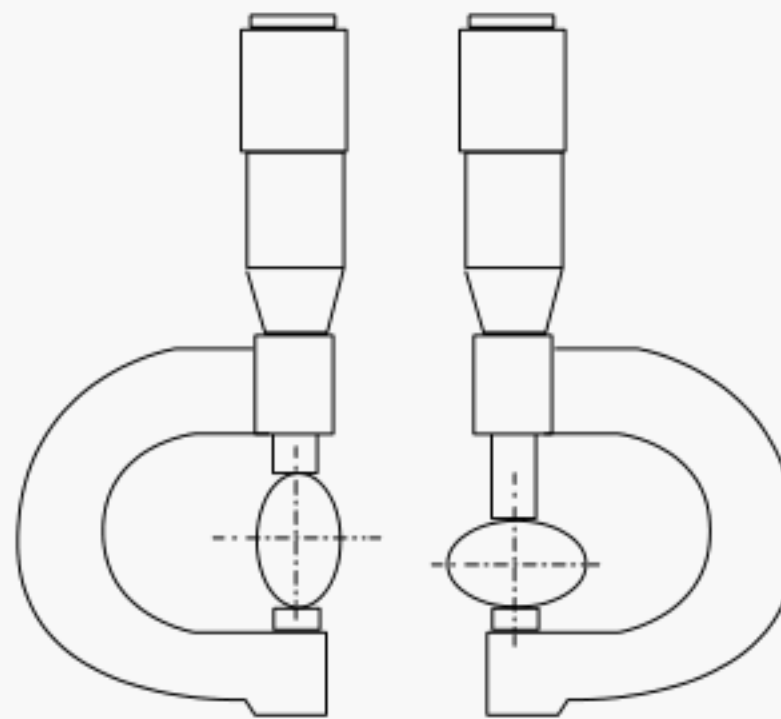


GENERAL NOTE: Possible form errors of the workpiece of Fig. B-2.1-1.



**Fig. B-2.3-2**

GENERAL NOTE: Out-of-straightness cannot be detected by two-point diameter measurements.

**Fig. B-2.3-3**

GENERAL NOTE: A micrometer will detect a two-lobe form error.

If the workpiece is not straight, a micrometer cannot detect this condition; see Fig. B-2.3-2. Note that a short ring gage also cannot detect 100% of the form error. In either case, an uncertainty component must be introduced in assessing conformity with the maximum diameter specification.

If the workpiece cross section is not round, a micrometer may or may not be able to detect the condition. If the cross section has an even number of lobes (for example, two, as shown in Fig. B-2.3-3), the error will be detected. If the cross section has an odd number of lobes (for example, three, as shown in Fig. B-2.3-4), the condition will not be detected and an uncertainty component must be introduced into the assessment of conformity with the actual local size specification.

Some workpieces have spiral lobes (so-called “barber poling”). If the pitch of the spiral is short, such lobing

may not be detected and an uncertainty component will be introduced.

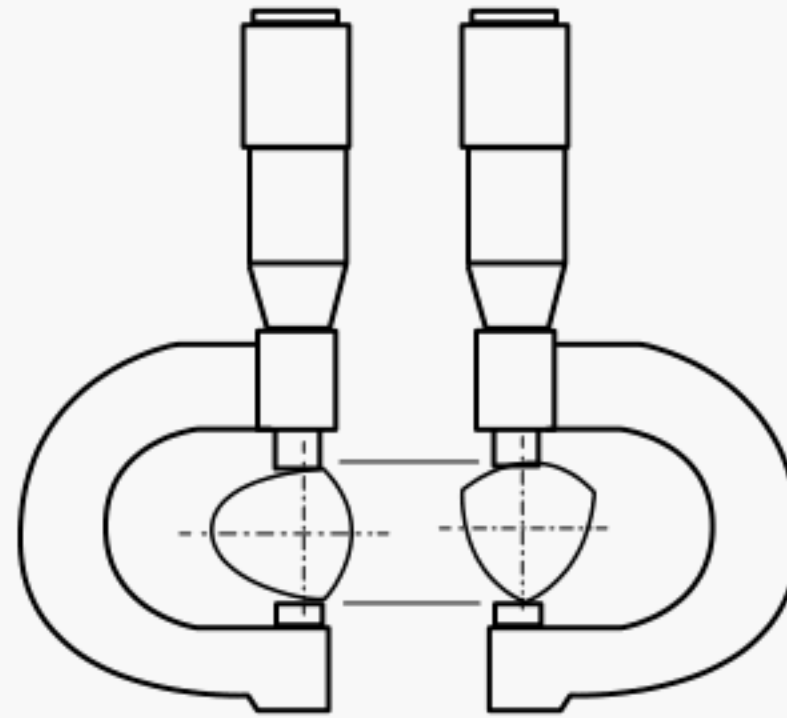
If the diameter is not constant along the length of the workpiece (for example, a tapered workpiece, as shown in Fig. B-2.3-5), the condition will be detected. It might be difficult, however, to determine the smallest value for the ALS, so that an uncertainty component would need to be introduced.

Many form errors are possible; Fig. B-2.3-6 shows some typical examples. Knowledge of likely form errors based on properties of the manufacturing process can help to guide the design of the measuring procedure to detect and remove nonconforming workpieces.

Similar analyses may be made for other types of measuring instruments and for other types of dimensional features.

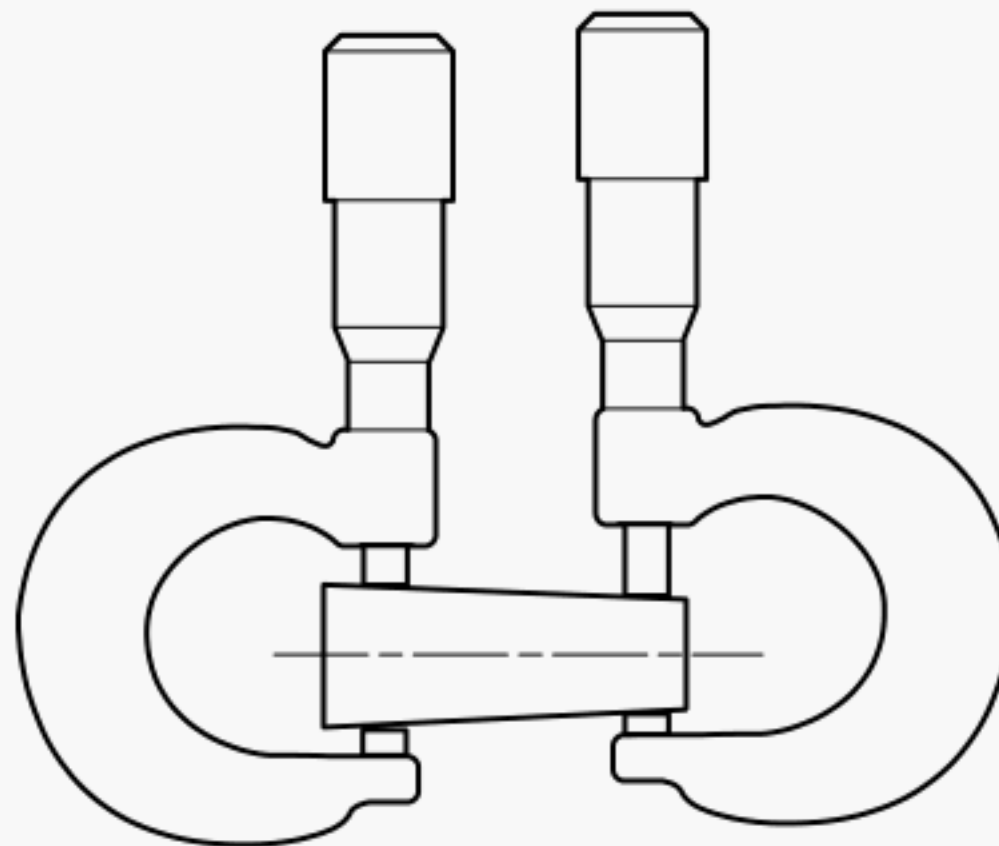


**Fig. B-2.3-4**



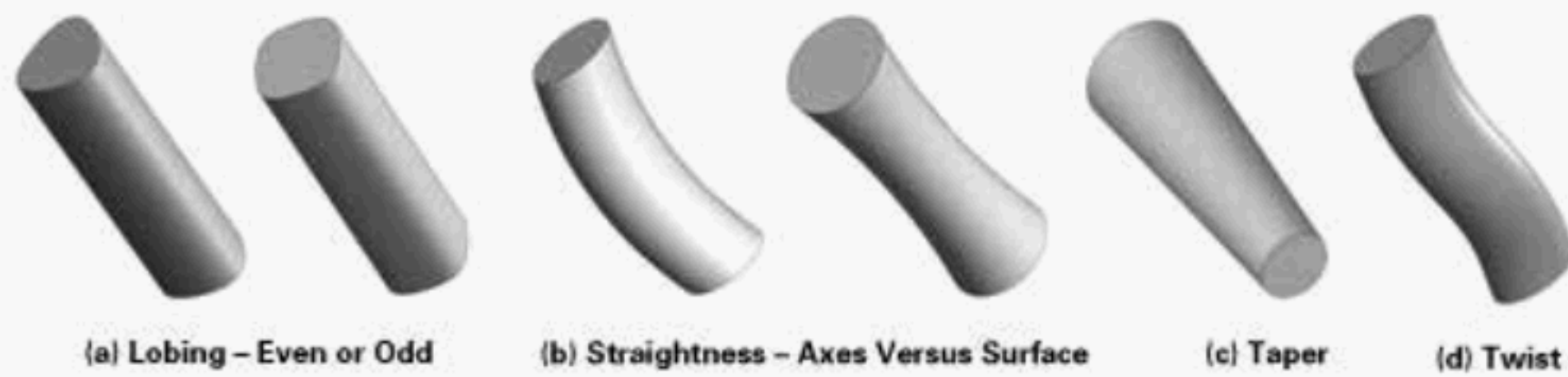
GENERAL NOTE: A three-lobe form error will not be detected.

**Fig. B-2.3-5**



GENERAL NOTE: A micrometer will detect a diameter variation.

**Fig. B-2.3-6**



GENERAL NOTE: Some possible form errors of cylindrical workpieces.



## NONMANDATORY APPENDIX C

### DIMENSIONAL MEASUREMENT UNCERTAINTY

#### C-1 PURPOSE

The purpose of this Nonmandatory Appendix is to assist the DMP in evaluating and applying measurement uncertainty.

#### C-2 BACKGROUND

Every dimensional measurement of a workpiece is characterized by two numbers: the measured value and its associated measurement uncertainty. The uncertainty is an indicator of measurement quality; in general, reducing the uncertainty will decrease the risks of pass and fail errors, for some increase in measurement cost.

The measured value is the result of the measurement. According to the GUM, the measurement is corrected for all recognized significant systematic errors; the measured value is then an (unbiased) estimate of the measurand. The expanded uncertainty is a measure of the interval about the estimate that is likely to contain the true value of the measurand. In the common case where the probability distribution for the measured quantity is Gaussian with a best estimate value,  $y$ , the true value lies in the interval  $[y - U_{k=2}, y + U_{k=2}]$  with a probability of about 95%, where  $U_{k=2}$  is the expanded uncertainty with coverage factor  $k = 2$ .

The authoritative guide to the evaluation and expression of measurement uncertainty is ANSI/NCSL Z540.2 [4], which is the U.S. Standard version of the GUM [2]. A condensed version is NIST Technical Note 1297 [12]. An introductory version of this document is provided by ASME B89.7.3.2 [13].

The general approach to the evaluation of measurement uncertainty consists of the following steps:

*Step 1:* Define the quantity intended to be measured, i.e., the measurand.

*Step 2:* Identify the significant influence quantities that can affect the outcome of the measurement.

*Step 3:* Develop a measurement model that relates the input quantities (composed of influence quantities) to the measurand (the output quantity).

*Step 4:* Assign probability distributions to the input quantities, yielding best estimates of their values and the associated standard uncertainties.

*Step 5:* Propagate knowledge of the input quantities, via the measurement model, into knowledge of the output quantity (the measurand).

*Step 6:* Calculate the estimate of the measurand, the associated combined standard uncertainty, and the expanded uncertainty with stated coverage factor.

For many dimensional measurements, the number of influence quantities is small and the problem of uncertainty evaluation can be simplified with respect to the general approach of the GUM. Such a simplified approach is described in ASME B89.7.3.2 [13], which is recommended as a basic guide for the DMP in designing and analyzing dimensional measurement procedures.

The following sections of this Nonmandatory Appendix discuss some of the influence quantities frequently encountered in dimensional measurements and approaches to dealing with them.

#### C-3 INFLUENCE QUANTITIES RELATED TO MEASURING PROCEDURE

##### C-3.1 Dimensional Measurement Planner Related

This section relates to discrepancies between requirements of the measurement task and characteristics of the chosen gaging method. Perhaps the most significant DMP-related source of error results from incomplete understanding of requirements per ASME Y14.5 or other GD&T standard.

Generally, the definition of a measurand suggests a gaging method. The DMP may choose a gaging method that does not fully comply with this principle either because no corresponding gage exists or because of nontechnical factors such as cost, time required, gage availability, or operator skill requirement. Such choice is permissible if the resultant measurement uncertainty is within acceptable limits.





### C-3.2 Attributes Measurements and Workpieces With Burrs

Measurements of features with burrs can cause some otherwise conforming workpieces to be rejected and some otherwise nonconforming workpieces to be accepted. Unless otherwise specified, the effects of burrs must be considered in all dimensional measurements and associated measurement uncertainties. If the manufacturing engineer desires to have burrs removed from the workpiece, it should be explicitly stated to the DMP to be included in the measurement plan, and if it is not stated, then all burrs must be included in the measurement.

### C-3.3 Coordinate Data Sets and Measurements Involving Form: Sampling Strategy

There are three areas of difficulty in accessing form from measured point data sets, such as those obtained using a coordinate measuring machine: estimating what happens between measured points, smoothing the data to reduce the effects of noise associated with the measurement process, and dealing with data judged to be invalid (outliers).

Three approaches are discussed as follows:

(a) The first approach is to fit a surface of perfect form (e.g., a perfect cylinder to represent a reamed hole) by the least-squares method. The least-squares method is a different measurand than that specified by ASME Y14.5, but it may be relatively accurate depending on the amount of measurement error relative to form error. Since a single measured value is used to represent both the AMS and ALS, uncertainty components can be assigned by considering plausible deviations of measured points from the fitted surface.

(b) The second approach is to fit a surface of perfect form to the extreme points of a data set. For example, to evaluate a drilled hole, a maximum inscribed cylinder fit is typically used; there are two obvious problems with this approach. First, if there are outliers in the data set, these are likely to be the points to which the cylinder is fitted. Second, no attempt is made to predict whether there are unmeasured points that lie inside the fitted cylinder.

(c) The third approach involves further work on the data points. Features of this approach might include

- (1) setting aside points with unusually large deviations as possible outliers
- (2) analyzing residuals from the fitted surface for systematic content, and modeling the systematic deviations
- (3) analyzing the possible outliers to determine if they can plausibly be considered to be valid data
- (4) analyzing the model with systematic deviations for compliance with specifications
- (5) assigning uncertainty components

### C-3.4 Aliasing in Point Data Sets

When a surface has a cyclic error, the distribution of sampled points can interact with the form error to give highly misleading results. Two particular examples are

(a) if the surface is sampled at intervals slightly different from the wavelength of the form error, the resulting data set can suggest a cyclic error very different from the actual form error.

(b) if the measured points are uniformly distributed with a period equal to the period of the form error (or a submultiple of this period), the resulting fitted feature can look perfect (neglecting measurement noise). A cylindrical bore with a pure eight-lobed form error, for example, will appear to have zero form error if sampled at eight points (or four points) uniformly distributed around the circumference. The diameter of the resulting fitted circle will depend on the phase of the sampled point distribution with respect to the actual form error.

The DMP must devise a point sampling plan that avoids these interaction problems, or assign a component of measurement uncertainty associated with such interactions.

For cylindrical surfaces, useful information regarding the interaction of measured point distributions with cyclic form errors, with a focus on measurement uncertainty, can be found in reference [14].

### C-3.5 Workpiece Distortion

Clamping and/or gravitational forces can distort a workpiece. If such problems are judged to be significant and cannot be avoided, estimates of their effects and associated uncertainty components must be assigned.

### C-3.6 Operator (Metrologist) Effects

The person performing the measurements should check gages and test setups for stability and proper functioning, read gage outputs accurately and objectively, and generally avoid problems that could significantly affect the quality of the measurements.

Particular attention should be given to avoiding parallax errors, applying gaging forces, the effects of body temperature on workpieces and measuring instruments, etc. An uncertainty component associated with operator-related measurement variation must be assigned if such variation is judged to be significant.

## C-4 GAGE-RELATED INFLUENCE QUANTITIES

### C-4.1 Attributes Gages

The uncertainty component associated with the gage is typically assigned based on the supplier's specifications or from calibration data. The condition of the gage relative to its condition at its last calibration will contribute to the associated uncertainty.





Where mastering of a gage is required, uncertainty components associated with the master and the mastering process must be assigned.

#### C-4.2 Variables Gages

**C-4.2.1 One-Dimensional Gages.** This classification includes micrometers, vernier and electronic calipers, height gages, air gages, displacement-measuring interferometers, etc. Associated uncertainty components can be assigned based on manufacturer's specifications and on calibration reports. Where mastering is required, gage resolution may contribute to the uncertainty of both mastering and measuring. Wear and corrosion of masters may require the assignment of additional uncertainty components.

**C-4.2.2 Two-Dimensional and Three-Dimensional Gages.** This classification pertains to CMMs and other gaging systems that produce point data sets.

Evaluation of measurement uncertainty for such systems, particularly CMMs, is an area under active development. A number of approaches have been taken; among them are the following:

(a) The first approach is based on the comparator principle. A calibrated master artifact that is similar to the workpiece is measured by the gaging system (e.g., a CMM). Differences between results of the measurements (performed under a variety of conditions) and calibrated values are used to estimate the CMM systematic error and the associated measurement uncertainty.

(b) A second approach realizes a virtual measuring system and is based on Monte Carlo sampling. A mathematical model is developed such that measurement errors may be calculated in terms of measured parametric errors (e.g., linear displacement errors, straightness, pitch, roll, yaw, and squareness errors). Probability distributions are assigned to the parametric errors, together with probing and dynamic errors, assumed workpiece form errors, etc. Workpiece measurement is simulated by repeated random draws from the probability distributions, yielding a numerical approximation for the probability distribution of the measurand. The combined standard uncertainty is calculated as a sample standard deviation.

(c) A third approach is evaluation of uncertainty components based on data recorded during system performance tests. Such tests are designed to be sensitive to components of the measuring system that can cause systematic errors and are thus sources of uncertainty. Thus, in principle, uncertainty components can be evaluated by working backwards from test results.

Repeatability and reproducibility test results can be part of this process. The general approach is to relate each workpiece measurement to a similar performance test measurement, and to adjust performance test results

for differences between the workpiece and performance test measurements.

An informative discussion of the problem of task-specific evaluation of measurement uncertainty using CMMs is given by reference [15].

### C-5 ENVIRONMENT-RELATED INFLUENCE QUANTITIES

#### C-5.1 General

For measurements of small workpieces with moderate tolerances under reasonable conditions, the contribution to measurement uncertainty due to environmental influences is usually negligible. For large gages and workpieces, or for small workpieces made to tight tolerances, environmentally related uncertainty components can be significant. The following discussion pertains to such cases.

#### C-5.2 Temperature: General Considerations

The general effects of temperature in dimensional measurement are discussed in detail in ASME B89.6.2 [16]. However, since the objectives of ASME B89.6.2 and ASME B89.7.2 are different, some interpretation is necessary. For example, ASME B89.6.2 advocates use of a thermal error index as a means for judging acceptability of a thermal uncertainty, while ASME B89.7.2 recommends combination of temperature-related uncertainty components with other contributors in order to determine the probabilities of pass and fail errors (consumer's and producer's risks).

By international agreement, dimensions and forms of workpieces are defined at the standard reference temperature of 20°C [17]. Thus if both a gage and a workpiece to be measured could be maintained at exactly 20°C, there would be no thermally related component of measurement uncertainty. Usually it is impractical to adequately approximate this condition. For temperatures other than 20°C, one must consider the effects of thermal expansion on both the gage and the workpiece.

Thermal effects in dimensional measurements, particularly differential expansion, are discussed in detail in ISO 16015:2003 [18] and in Nonmandatory Appendix D of ASME B89.7.3.2-2007 [13]. The basic ideas are briefly reviewed below.

**C-5.2.1 Constant Temperature: Differential Expansion.** Consider the basic two-point measurement shown in Fig. C-5.2.1-1, where a micrometer is used to measure the length of a simple workpiece. Assume that both the micrometer and the workpiece are at a uniform temperature,  $T$ , other than 20°C.

The measurand is the length,  $L_w$ , of the workpiece at 20°C; at temperature  $T$ , the length is

$$L_w(1 + \alpha_w \Delta T) \quad (C-1)$$





Fig. C-5.2.1-1



GENERAL NOTE: A micrometer measures the length of a workpiece at a constant temperature,  $T$ .

where  $\alpha_w$  is the workpiece coefficient of thermal expansion (CTE) and  $\Delta T = T - 20^\circ\text{C}$ .

The micrometer reads  $L_m$ , which, for a well calibrated instrument, would be the spacing between the anvils at  $20^\circ\text{C}$ ; at temperature  $T$  the anvil spacing is

$$L_m(1 + \alpha_m \Delta T) \quad (\text{C-2})$$

where  $\alpha_m$  is the micrometer CTE.

In the length measurement, the micrometer anvils are in contact with the ends of the workpiece so that eqs. (C-1) and (C-2) are equal. Thus

$$L_w(1 + \alpha_w \Delta T) = L_m(1 + \alpha_m \Delta T) \quad (\text{C-3})$$

Solving for  $L_w$  and neglecting terms of second order (see ASME B89.7.3.2, Nonmandatory Appendix D), the desired length is

$$L_w = L_m + L_m(\alpha_m - \alpha_w) \Delta T \quad (\text{C-4})$$

The second term on the right side of this equation is a correction for differential expansion between the gage (micrometer) and the workpiece. It is seen that if the micrometer and the workpiece have the same CTE (i.e., a steel micrometer measuring a steel block), then the correction for differential expansion will be zero for any constant temperature,  $T$ .

On the other hand, if the gage and workpiece have different CTEs or different temperatures, and if a correction is not calculated and applied to account for the difference, then the systematic error (bias) due to thermal effects will be just the difference in thermal expansion over the length,  $L_m$ , as given by the correction term in eq. (C-4).

**C-5.2.1.1 Uncertainty Evaluation for Differential Expansion.** Even for a gage and workpiece with the same nominal CTE, although the correction for expected differential expansion is zero, there will still be uncertainty components associated with inexactly known CTEs and ambient temperature,  $T$ .

The standard uncertainty associated with the measured length,  $L_w$ , of the workpiece follows by propagation

of uncertainty as described in the GUM with the measurement model given by eq. (C-4). The component  $u^2(L_m)$  arising from micrometer calibration will be ignored in the following, in order to focus on the component  $u^2_T(L_w)$  arising from thermal effects (see Nonmandatory Appendix D of ASME B89.7.3.2).

Setting  $u^2(L_m) = 0$  and defining  $\Delta\alpha = \alpha_m - \alpha_w$ , the result is

$$u^2_T(L_w) = (L_m)^2 \left[ (\Delta T)^2 [u^2(\alpha_m) + u^2(\alpha_w)] + (\Delta\alpha)^2 u^2(\Delta T) \right] \quad (\text{C-5})$$

Coefficients of thermal expansion typically exhibit variations of  $\pm 10\%$  about their nominal values. An uncertainty component such as  $u^2(\alpha_m)$  is then evaluated by assigning a uniform probability distribution of appropriate width. The temperature-related component  $u^2(\Delta T) = u^2(T)$  is evaluated based on available information about the thermal environment during the measurement.

#### C-5.2.1.2 Special Case: Equal CTEs and $T = 20^\circ\text{C}$ .

If the gage and workpiece have the same CTE (e.g., a steel micrometer used to measure a steel workpiece), and the best estimate of the temperature is  $20^\circ\text{C}$  [e.g., measurement lab kept at  $(20 \pm 1)^\circ\text{C}$ ], then  $\Delta\alpha = \Delta T = 0$ , and from eq. (C-5) it is seen that  $u^2_T(L_w) = 0$ . It seems reasonable that no correction need be applied for differential expansion when  $\alpha_w = \alpha_m$ , but it does not seem reasonable that the associated standard uncertainty should be independent of uncertainties in the CTEs, the temperature, and the length of the workpiece.

The GUM propagation of uncertainty approach is a first-order linear approximation that fails in this case. The correct solution requires the inclusion of second-order terms in the expansion of the measurement equation (see GUM, sections 5.1.2 and H.1.7); the resultant combined standard uncertainty in the present example is

$$u_T(L_w) = \sqrt{2} L_m u(\alpha) u(\Delta T) \quad (\text{C-6})$$



where  $u(\alpha) = u(\alpha_w) = u(\alpha_m)$ . Because it depends on a product of standard uncertainties, the resultant thermal component of length uncertainty may be negligible compared to the effects of other influence quantities, but it must be considered in case of very tight tolerances or in the case where no thermal compensation is applied and, although the average room temperature is 20°C, the temperature variation is significant.

NOTE: Limitations due to the linear approximation of the GUM propagation of uncertainty approach can be avoided by propagation of distributions using Monte Carlo sampling; see, for example, Supplement 1 to the GUM, JCGM 101:2008; also see ASME B89.7.3.2, section D-3 for a simplified method of estimating this effect.

**C-5.2.1.3 Other Considerations at Constant Temperature.** If the gage and/or the workpiece is made up of two or more components having different coefficients of thermal expansion, the structure must be carefully analyzed to determine if there will be additional uncertainty components due to bending or twisting.

**C-5.2.2 Slowly Changing Temperature.** Since the temperature change in a body is caused by heat flow and heat flow is caused by temperature gradients, changes in temperature are always accompanied by temperature gradients. A slowly changing temperature is one for which gradients are small, and thermal expansions are approximately the same as they would be if the temperature were uniform at all points in the body at a particular time.

Situations where temperature is slowly varying are characterized by one or more of the following:

- slowly varying environment temperature
- high thermal conductivity
- short heat flow paths
- low specific heat
- low density
- low heat transfer to the environment

The following equations are approximations that are useful for estimating gage and/or workpiece temperatures when temperature change is slow.

Consider a workpiece in equilibrium with the ambient air at temperature  $T_0$ . At time  $t = 0$ , a step change in temperature, from  $T_0$  to  $T_0 + \Delta T$ , occurs, perhaps by moving the workpiece to another location. Then the temperature of the workpiece at a later time,  $t$ , will be given by

$$T(t) = T_0 + \Delta T(1 - e^{-t/\tau}) \quad (\text{C-7})$$

where  $\tau$  is a time constant that depends on the workpiece and the heat transfer environment. The exponential relaxation form of eq. (C-7) is known as Newton's law of cooling (or heating, depending on the sign of  $\Delta T$ ). The time constant,  $\tau$ , can be determined experimentally

as the time required for the workpiece temperature to change by 63.2% of the step size  $\Delta T$ .

Estimating the temperature of a gage and/or a workpiece in a time-varying thermal environment can be a challenging problem. The following discussion presents some useful information on the thermal behavior of materials that can aid in such estimation.

The thermal response of a material to a change in temperature is governed by a material parameter,  $D$ , called the thermal diffusivity and defined by

$$D = \frac{\kappa}{\rho c} \quad (\text{C-8})$$

where  $\kappa$  is the thermal conductivity,  $\rho$  the mass density, and  $c$  the specific heat capacity.

The steady-state response of a semi-infinite piece of material exposed to a sinusoidal temperature variation at its boundary is a sinusoidal function of time with a phase shift and amplitude that depend on distance from the boundary. The amplitude of the temperature variation falls off exponentially with distance, with a characteristic length (or penetration length), given by

$$x_c = \sqrt{\frac{DP}{\pi}} \quad (\text{C-9})$$

where  $P$  is the period of the driving temperature sinusoid. From this result is seen that at any depth in the material the response scales like the square root of the period (or the reciprocal of the square root of the driving frequency). Two approximate limits can be identified by comparing the typical dimensions of a workpiece with the penetration length,  $x_c$ .

(a) If the workpiece is large compared with  $x_c$ , then the bulk of the workpiece material will not experience a significant temperature oscillation. In this case, the workpiece temperature will be approximately equal to the average temperature of the environment.

(b) If the workpiece is small compared with  $x_c$ , then all points within the workpiece material will essentially follow the sinusoidal temperature oscillation. In this case, the workpiece temperature will be approximately equal to the instantaneous temperature of the environment.

This simple model assumes that the workpiece environment can rapidly supply or absorb thermal energy from the workpiece with negligible change in the environment temperature. This would be a good approximation for a small workpiece in a water bath, but a very poor approximation for a large workpiece in still air.

If the workpiece temperature is a significant source of measurement uncertainty, knowledge of the temperature must be characterized by an appropriate probability distribution. In (b) above, for example, if all that is known of the thermal environment are the limits of the temperature oscillation, a U-shaped distribution can be assigned to model knowledge of the workpiece temperature (see Nonmandatory Appendix B of ASME B89.7.3.2).





Thermal response to air temperature can be modified by conduction via contact between materials. Consider, for example, a gage and a workpiece on a steel surface plate. If the surface plate is relatively massive, its temperature variation with respect to changes in air temperature will be relatively small. Thus conduction between the surface plate and the gage and workpiece will tend to stabilize their temperatures. This can cause the temperature difference between the gage and the workpiece to either increase or to decrease, depending on how much conduction (dependent on the area of contact) occurs between the surface plate and the gage and workpiece.

Convective heat flow is usually predominant. The temperature should be calculated based on guidelines given above. Assigned temperatures should be modified, based on the judgment of the DMP, for the effects of conduction and radiation (see para. C-5.2.4). For workpieces on large metal surfaces, for example, conduction will dominate; for workpieces on insulating materials (e.g., wood) in a forced air environment, convection will dominate. In general, both effects can be significant.

**C-5.2.3 Rapidly Changing Temperature.** If small, rapid air temperature fluctuations occur (due to air conditioning, for example), and if the gage and workpiece are relatively massive, the effects of the fluctuation may be negligible. This is because not enough heat flows into and out of the gage and/or workpiece to cause significant internal temperature changes.

For solid metal workpieces in air, the internal distribution of thermal energy is much faster than the rate it can be absorbed. The internal temperature of such a workpiece cannot vary much because, with air coupling, energy can only be slowly transferred to the workpiece so that its internal temperature is nearly uniform. The penetration depth for steel — even for a frequency as short as a few minutes — is equal to the full diameter of the pin example in Nonmandatory Appendix A, so that the energy will rearrange itself in such a way as to yield a constant, homogeneous pin temperature.

Large step-function changes in air temperature can cause significant temperature gradients in the gage and workpiece when these are made of poorly thermally conducting materials, e.g., granite, or when the object contains numerous internal cavities resulting in a thin-walled material that cannot internally transport large amounts of energy. Such gradients can cause bending and twisting. If such situations cannot be avoided, uncertainty components must be assigned, based on experience or direct experimental measurements, to account for the effects of temperature gradients. Theoretical models to predict these effects are complex and difficult and can depend on detailed knowledge of issues such as joint connections and weldments.

**C-5.2.4 Other Thermal Effects.** Radiant energy can be a significant contributor to measurement uncertainty with large gages and workpieces. The major sources of radiant energy are sunlight and artificial lighting. It is good practice to minimize radiant effects before evaluating their associated uncertainty components.

Because of its strength and variability, direct sunlight should be avoided. For artificial lighting, radiant energy outside the visible spectrum should be minimized. Indirect lighting is often effective. Lighting should be as uniform and indirect as possible to minimize uneven heating of the gage and workpiece.

Evaluation of the associated uncertainty components will usually be based on judgment and experience. Radiant energy can typically cause temperature differences of a fraction of a degree in different parts of a gage or workpiece. One of the most common problems caused by radiant energy is change in flatness of large granite surface plates.

A significant source of error is measurement of workpieces that have not been thermally stabilized (soaked out) in the inspection environment. The associated uncertainty components can be evaluated using the formula for temperature step response, eq. (C-7).

Air from a heating or cooling duct blowing directly on a gage or workpiece can be a significant source of thermally related measurement error. The effect is larger than might be expected because high-velocity air increases the heat transfer coefficient. It is usually easier to eliminate this problem than to model its effect on the measurement.

A frequently encountered problem is heating of the gage and/or workpiece caused by handling. This topic falls more under the heading of good metrology practice than under thermal uncertainty.

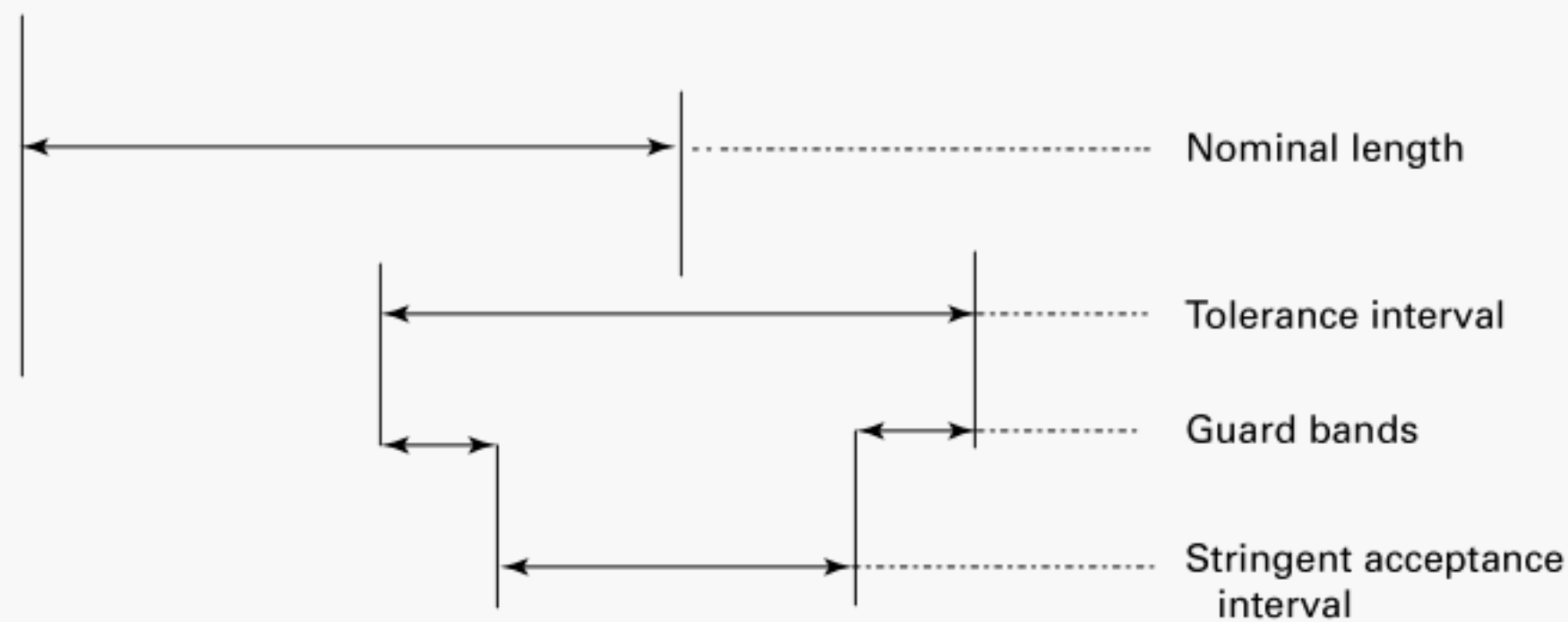
Spatial variations of air temperature can create uncertainties in the measurement of large workpieces. The effects of such variations can be estimated by means of mathematical models, judgment, and experience.

In some optical gaging systems (such as displacement-measuring interferometers), the effects of air temperature, pressure, relative humidity, and composition on the refractive index of air can be significant. Methods for evaluating components of measurement uncertainty associated with such effects are typically provided in manufacturer's operating manuals supplied with these gaging systems.

In gages having hydrostatic air bearings, two problems may arise. First, any variation of supply air temperature may cause a variation in gage temperature. Such variation is easily avoided by ensuring sufficient length of air line in the measuring room to ensure that the supply air comes to room temperature. The second problem arises from throttling of air through the bearing, which causes local cooling of the gage via the Joule-Thompson effect. The effects of such throttling are typically allowed for in the gage performance specifications.



Fig. C-6-1



GENERAL NOTE: A stringent acceptance interval, defined by gaging limits inside the tolerance limits, will reduce the probability of a pass error.

### C-5.3 Vibration

The characteristic of vibration that can cause systematic measurement error is relative motion between the gage sensing or contacting element and the measured point on the workpiece. In what follows, such relative motion is referred to simply as vibration. A systematic error can occur, for example, when using a CMM that records a “snapshot” of workpiece contact at some particular instant of time. For probing systems such as typical LVDT gage heads or capacitance gages, which have an averaging characteristic, the uncertainty component associated with vibration is usually negligible.

### C-5.4 Other Environmental Effects

While temperature and vibration are usually the major influence quantities associated with environmental effects, the measurement task should be reviewed to determine if other effects may contribute significantly to the uncertainty. Examples of such effects include magnetic fields, gravity, acoustic noise, electronic drift, barometric pressure, humidity, electromagnetic interference, and contamination (e.g., dirt, oil, etc.).

## C-6 MEASUREMENT UNCERTAINTY AND ACCEPT/REJECT DECISIONS

As an example of the role of measurement uncertainty in decision making, consider a length-measurement task where a nominal (design) length and a symmetrical bilateral (plus or minus) tolerance limit are given.

Following a dimensional measurement plan, the workpiece is measured, yielding a measured length (the estimate) and an associated standard uncertainty, evaluated as discussed in the previous sections. A workpiece having a true length outside the tolerance interval is nonconforming. A decision to accept or reject the workpiece is based on the measured value in relation to

an acceptance interval defined by a pair of gaging (or acceptance) limits.

The role of measurement uncertainty in the formulation of decision rules and associated documentation requirements are well described in ASME B89.7.3.1 [19]. The most common form of an accept/reject decision rule is called simple acceptance, in which a feature of length is accepted as conforming if its measured value lies in the tolerance interval, and is rejected otherwise. Simple acceptance is typically accompanied by a requirement that the measurement uncertainty be no larger than a stated fraction of the tolerance interval (see ASME B89.7.3.1, para. 4.1).

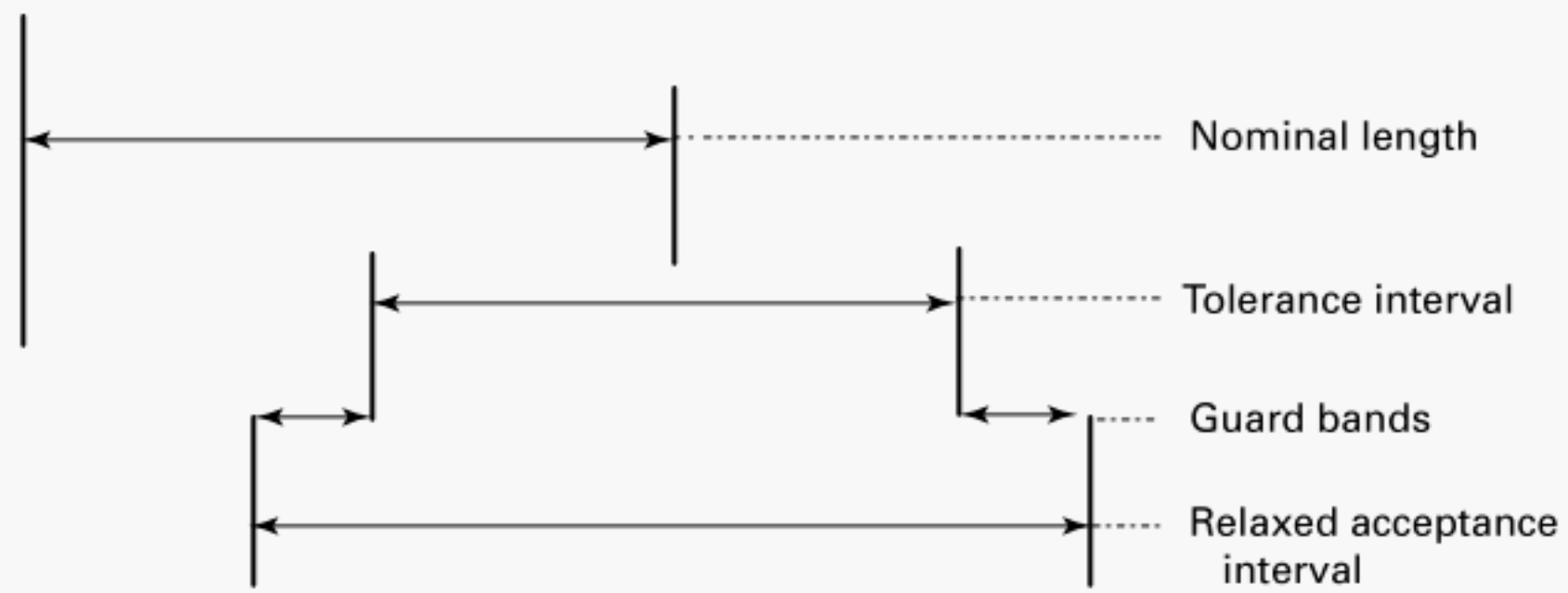
When a measured length is within tolerance but close to one of the tolerance limits, the probability that the true length lies outside the tolerance interval can be significant. Accepting such a nonconforming workpiece is called a pass error, and the probability of such an error is called consumer’s risk. Figure C-6-1 shows a stringent acceptance interval that will reduce pass errors. The gaging limits are offset from the tolerance limits by guard bands whose widths are expressed as fractions of the  $k = 2$  expanded uncertainty,  $U$  (see ASME B89.7.3.1, para. 4.2).

When a measured length is out of tolerance but close to one of the tolerance limits, the probability that the true length lies inside the tolerance interval can be significant. Rejecting such a conforming workpiece is called a fail error, and the probability of such an error is called producer’s risk. Figure C-6-2 shows a relaxed acceptance interval that will reduce fail errors. In this case, the gaging limits lie outside of the tolerance limits (see ASME B89.7.3.1, para. 4.3).

The gaging limits are set so as to yield acceptable probabilities of pass and fail errors. These probabilities (consumer’s and producer’s risks) form the subject of ASME B89.7.4.1 [20] and JCGM #106 [27], and are discussed in Nonmandatory Appendix D. If calculated probabilities are unsatisfactory, new gaging (acceptance) limits must be chosen or steps taken to change the process distribution or the measuring system.



**Fig. C-6-2**



GENERAL NOTE: A relaxed acceptance interval, defined by gaging limits outside of the tolerance limits, will reduce the probability of a fail error.



# NONMANDATORY APPENDIX D

## PROBABILITIES OF PASS AND FAIL ERRORS

### D-1 INTRODUCTION

This Nonmandatory Appendix concerns dimensional measurement of workpiece properties. Every measurement has an associated uncertainty, so that near the limits of acceptability (i.e., tolerance limits) it is not possible to determine unequivocally whether a workpiece property conforms or not. Accept/reject decisions are thus matters of probability.

In such circumstances, when a decision is made to accept or reject a workpiece, there is a possibility of an incorrect decision (i.e., a pass or fail error). This Nonmandatory Appendix presents methods for calculating the probabilities of such errors, following the technical approach of ASME B89.7.4.1 [20].

Two cases are considered

(a) two-sided measurements of properties such as length or diameter, which can fail to conform with specification because they are too large or too small (see Fig. D-1-1)

(b) one-sided measurements of properties such as flatness or straightness, which can fail to conform only when they are too large (see Fig D-1-2)

Despite the many possible relationships between the measured value of a quantity, tolerance limits, gaging limits, process distribution, and measurement uncertainty, determination of the probabilities of pass and fail errors is a straightforward exercise, once a suitable model of the production process and the measuring system have been developed.

The calculations involve integrations of probability distributions that usually cannot be evaluated in closed form. Such integrations, however, are readily performed numerically, to any desired degree of approximation, by commercially available mathematical software. Development of the relevant process and measurement models and calculation of the desired probabilities are illustrated in the following sections.

### D-2 TOLERANCE AND CONFORMANCE

A central concept of this Nonmandatory Appendix is that a dimensional property whose true value lies in the tolerance interval is conforming, and is nonconforming otherwise. The designations “conforming” and “nonconforming” are used with respect to the definitions of pass and fail errors.

### D-3 TRUE VALUES AND MEASURED VALUES

Conformance and nonconformance are attributes of true values of dimensions of interest, which by their nature cannot be exactly known. Accept and reject decisions are based on observed measured values and their locations with respect to assigned gaging limits (see Figs. D-1-1 and D-1-2). In calculating consumer’s and producer’s risks, true values and measured values are modeled by random variables,  $X$  and  $X_m$ , with possible values,  $x$  and  $x_m$ , respectively.

#### D-3.1 Possible Outcomes of an Accept/Reject Decision

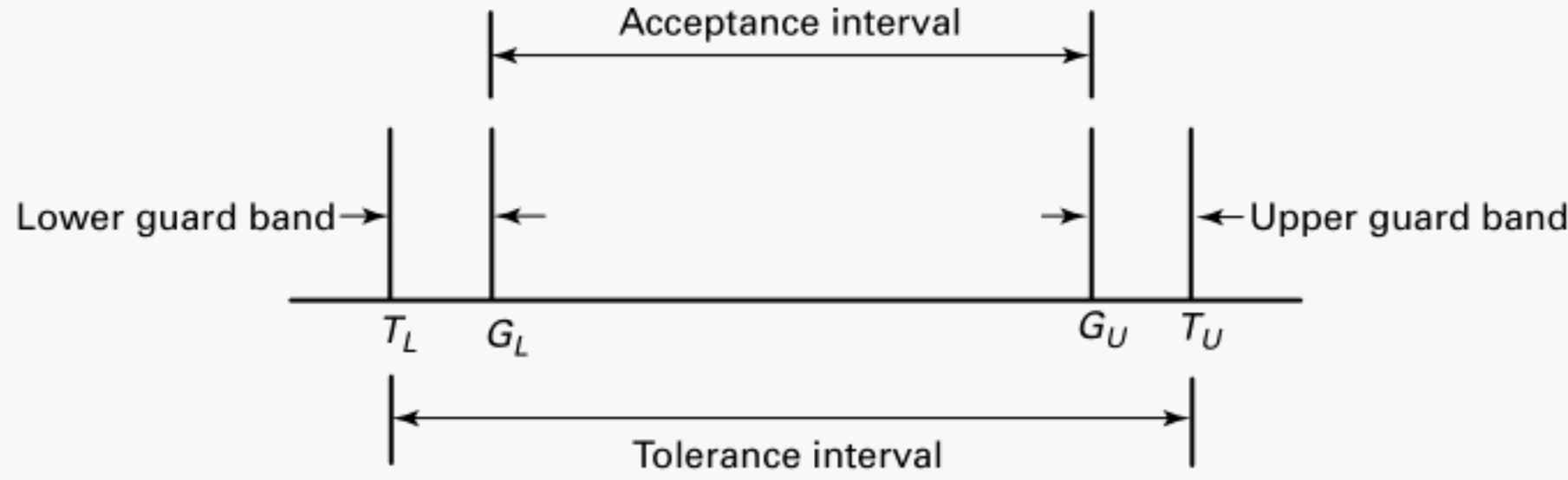
This Nonmandatory Appendix deals with a simple binary decision rule in which there are only two possible actions: accept a measured dimension as conforming or reject it as nonconforming. In this case, the possible outcomes following a measurement are illustrated in Fig. D-3.1-1.

In general, there may be more than two possible actions specified in a decision rule. In particular, when there is a transition zone between a tolerance limit and an acceptance limit (defined by a guard band), it might be desirable to have an alternative to simple acceptance or rejection for a measured value lying in such a zone (see ASME B89.7.3.1 [19], section 4.4).



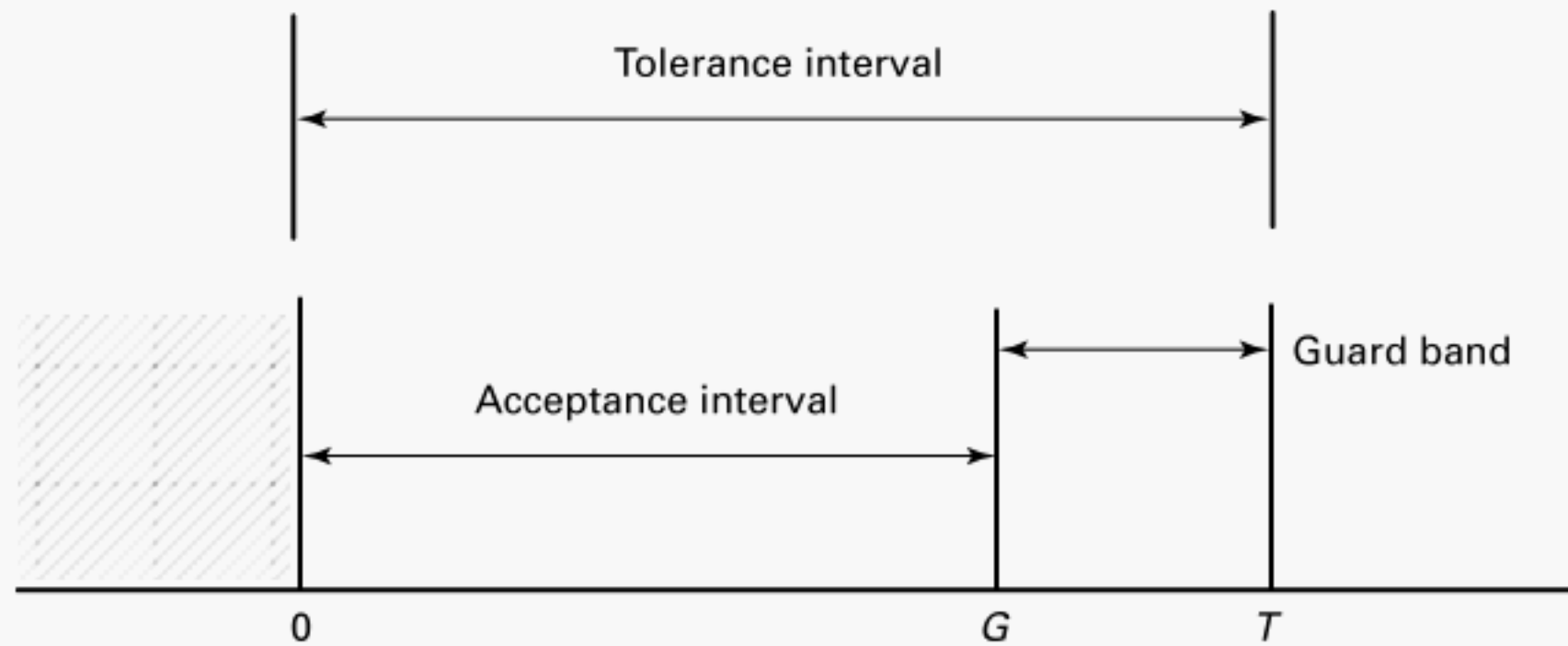


Fig. D-1-1



GENERAL NOTE: A two-sided measurement of a property such as the length of a workpiece. The true length is specified to lie in a tolerance interval defined by limits ( $T_L$ ,  $T_U$ ). The workpiece is accepted as conforming if its measured length lies in an acceptance interval defined by gaging limits ( $G_L$ ,  $G_U$ ), and rejected as nonconforming otherwise. Shown is a stringent acceptance interval, per ASME B89.7.3.1.

Fig. D-1-2



GENERAL NOTE: A one-sided measurement of a measurable property such as flatness, straightness, sphericity, etc. The tolerance interval is defined by a maximum allowed value,  $T$ . The property is accepted as conforming if its measured value lies in an acceptance interval defined by a gaging limit,  $G$ , and rejected as nonconforming otherwise. Shown is a stringent acceptance interval, per ASME B89.7.3.1.

### D-3.2 Joint Probability Distribution for $X$ and $X_m$

Consider a workpiece chosen at random from the production process, and a dimensional property of the workpiece subsequently measured in order to decide conformance with specification. Belief in the possible true values,  $X$ , of the dimension, and possible measured values,  $X_m$ , produced by the measuring system are characterized by a joint probability density function (PDF)  $f(x, x_m)$ .

Given the joint PDF  $f(x, x_m)$  for the randomly chosen workpiece, the probability that the true value,  $X$ , lies in the interval  $[a, b]$  and the measured value,  $X_m$ , lies in the interval  $[c, d]$  is given by

$$\Pr(a \leq X \leq b \text{ and } c \leq X_m \leq d) = \int_c^d \int_a^b f(x, x_m) dx dx_m \quad (\text{D-1})$$

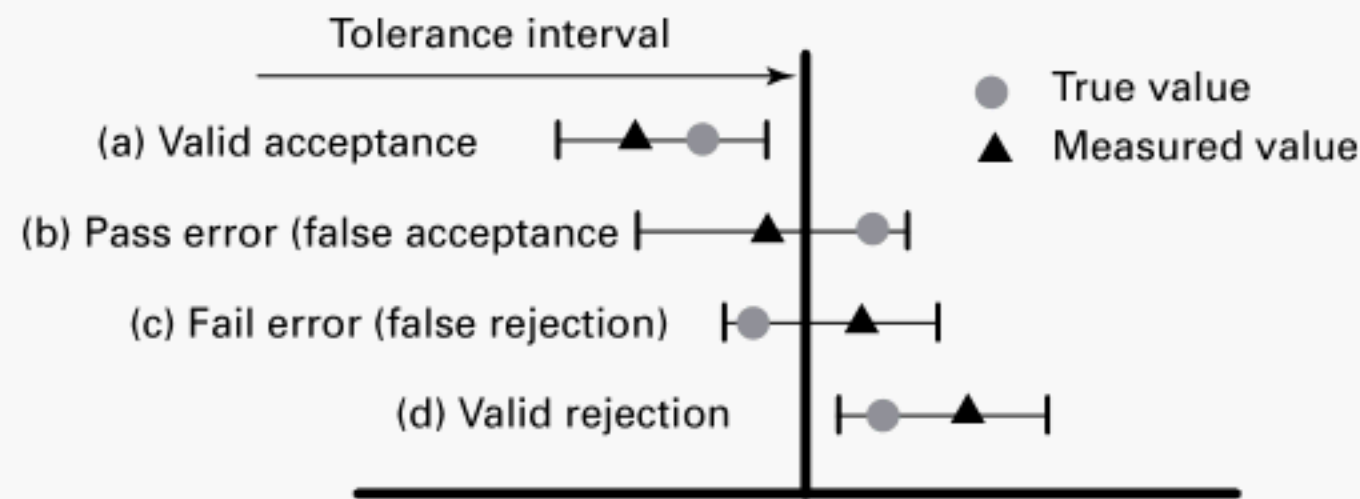
Equation (D-1) is the basic formula used in calculating probabilities of pass errors and fail errors, given particular values for the integration limits. In a two-sided measurement with a measured value within the gaging limits, for example,  $c = G_L$  and  $d = G_U$ .

The joint PDF  $f(x, x_m)$  depends on knowledge of the production process and the measuring system. The form of the dependence is written as

$$f(x, x_m) = p_0(x)p(x_m | x) \quad (\text{D-2})$$



Fig. D-3.1-1



GENERAL NOTE: Simple acceptance ( $G_U = T_U$ ) decision rule near an upper tolerance limit, with four 95% coverage intervals. Decisions to accept or reject inspected items are based on measured values (triangles); the true values (circles) cannot be known. Illustrations (b) and (c) lead to incorrect decisions called pass errors and fail errors, respectively.

In this equation,  $p_0(x)$  is the distribution of possible true values of the dimension, which depends on the production process and is independent of the measuring system.

The quantity  $p(x_m | x)$  characterizes the measuring system; it is a conditional probability density that encodes and conveys belief in a possible measured value,  $x_m$ , that might be observed when measuring a dimension with true value  $X = x$ , assumed to be known.

NOTE: The form of eq. (D-2) follows from a general result in probability theory called the product rule.

Calculation of the probabilities of pass errors and fail errors then requires assigning the distributions  $p_0(x)$  and  $p(x_m | x)$ ; setting the gaging limits; and using the basic formula of eq. (D-1) together with eq. (D-2). These steps are discussed in what follows, together with example calculations.

#### D-4 PROCESS DISTRIBUTION, $p_0(x)$

The characteristics of the manufacturing process are typically studied by measuring a sample of its output. Based on these measurements and other relevant information, such as experience with similar processes, a PDF  $p_0(x)$  is assigned to describe and encode belief in the possible values of a dimensional property of interest, before performing a measurement. This PDF is called the process distribution.

In many cases, the process is well represented by a Gaussian (normal) distribution

$$p_0(x) = \frac{1}{u_p \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x - x_0}{u_p} \right)^2 \right] \quad (D-3)$$

where  $x_0$  is the mean of the distribution (and the most probable value of  $X$ ) and  $u_p$  is the standard deviation.

NOTE: Since  $p_0(x)$  characterizes knowledge of the true value of a dimensional property prior to measurement, it is often referred to as the prior distribution.

In the case where  $p_0(x)$  is assigned based on measurement of a large sample of workpieces, the standard deviation,  $u_p$ , will typically be equal to an experimentally determined sample standard deviation.

For two-sided measurements with upper and lower tolerance limits, the process is often adjusted so that the average produced dimension,  $x_0$ , lies at the midpoint of the tolerance interval. In some cases it might be desirable to bias the process toward one of the limits. A process to produce a dimensional spacer, for example, might be biased toward the upper tolerance limit in order to reduce the number of workpieces that are too short and cannot be reworked. The DMP is responsible for adjusting the process in order to achieve an acceptable distribution of produced workpieces.

#### D-5 MEASURING SYSTEM DISTRIBUTION, $p(x_m | x)$

The result of a measurement of a dimension of interest is summarized by a measured value,  $x_m$ , and an associated standard uncertainty,  $u_m$ . The measuring system is modeled by considering the distribution of measured values that



might be realized when measuring a dimension whose true value,  $X = x$ , is assumed to be known. Such a distribution is called the measuring system distribution.

When the measurement is corrected for all recognized significant systematic effects via appropriate calibrations (see GUM [2], section 3.2.4), then the most probable measured value would just be  $x_m = x$ . The dispersion of probable measured values,  $x_m$ , about the (assumed known) true value is characterized by the standard uncertainty,  $u_m$ , evaluated according to the guidelines described in the GUM.

For many types of measurements, the measuring system is well characterized by a Gaussian distribution

$$p(x_m | x) = \frac{1}{u_m \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x_m - x}{u_m} \right)^2 \right] \quad (\text{D-4})$$

Other distributions are possible, depending on what is known about the measurement process (e.g., a  $t$ -distribution when the result is an average of a few noisy instrument readings). It is the responsibility of the DMP to assign the PDF  $p(x_m | x)$  that reasonably describes the behavior of the measuring system.

For measurements that are not corrected for known systematic effects and thus contain a known systematic error (or bias),  $b$ , the most probable measured value is obtained by replacing  $x$  with  $x + b$  in the equation for the measuring system distribution.

NOTE: In Nonmandatory Appendix D of the first edition of this Standard (ASME B89.7.2-1999) a measurement error distribution is used, rather than a measuring system distribution as described above. Since the error,  $E$ , is defined by  $E = x_m - X$ , possible values  $e$  of the error when measuring a dimension  $X = x$ , assumed to be known, are given by  $e = x_m - x$ .

From eq. (D-4) it follows that the corresponding Gaussian PDF for possible values of the measurement error is just

$$p(e | x) = \frac{1}{u_m \sqrt{2\pi}} \exp \left( -\frac{e^2}{2u_m^2} \right)$$

so that the best estimate of the error is zero in a measuring system free from bias. The two approaches give identical results for the probabilities of pass and fail errors, involving only a shift in limits of integration.

## D-6 GAGING LIMITS

When a workpiece dimension is measured using a binary decision rule, the dimension is accepted as conforming if the measured value is within the gaging limits and rejected otherwise.

The DMP may change the gaging limits to adjust the probabilities of pass and fail errors (consumer's and producer's risks). Bringing the gaging limits in toward the nominal dimension will decrease the probability of a pass error (accepting a nonconforming workpiece) and increase the probability of a fail error (rejecting a conforming workpiece). Moving the gaging limits out has the opposite effect. It's essential to note the decision related to level of risk is a business decision.

## D-7 NOTATION

Calculation of the probabilities of pass and fail errors is characterized by an unknown true value of a dimension, an observed measured value, and the following inputs which must be supplied by the DMP:

- (a) the process distribution
- (b) the measuring system distribution
- (c) the tolerance limits
- (d) the gaging limits

The following symbols are used in the calculations:

$f(x, x_m)$	=	joint distribution = $p_0(x)p(x_m   x)$
$G$	=	gaging limit for a one-sided measurement
$g$	=	guard band for a one-sided measurement
$G_L, G_U$	=	lower and upper gaging limits for a two-sided measurement
$g_L, g_U$	=	lower and upper guard bands for two-sided measurements
$p(x_m   x)$	=	measuring system distribution
$p_0(x)$	=	process distribution



$T$	=	tolerance limit for a one-sided measurement
$T_L, T_U$	=	lower and upper tolerance limits for a two-sided measurement
$x$	=	possible true values of a dimension
$x_m$	=	possible measured values

## D-8 TWO-SIDED MEASUREMENTS

Figure D-1-1 shows the tolerance and acceptance intervals for a two-sided measurement. Calculation of the desired probabilities will be illustrated using Gaussian process and measuring system distributions. Using eqs. (D-3) and (D-4), the joint PDF  $f(x, x_m)$ , eq. (D-2), is written explicitly as a bivariate Gaussian distribution

$$f(x, x_m) = p_0(x)p(x_m | x) = \frac{1}{2\pi u_p u_m} \exp \left\{ -\frac{1}{2} \left[ \left( \frac{x - x_0}{u_p} \right)^2 + \left( \frac{x_m - x}{u_m} \right)^2 \right] \right\} \quad (D-5)$$

This joint distribution is used as the integrand in eq. (D-1) using appropriate limits of integration.

### D-8.1 Probability of a Pass Error: Consumer's Risk, $R_c$

A pass error occurs when measurement of a nonconforming workpiece results in a measured value within the gaging limits. For such an incorrect decision,  $X < T_L$  or  $X > T_U$  and  $G_L \leq X_m \leq G_U$  and the integral over  $x$  in eq. (D-1) thus consists of two parts: one from  $-\infty$  to  $T_L$  and another from  $T_U$  to  $\infty$ .

The consumer's risk is therefore given by

$$R_c = \int_{G_L}^{G_U} \int_{-\infty}^{T_L} f(x, x_m) dx dx_m + \int_{G_L}^{G_U} \int_{T_U}^{\infty} f(x, x_m) dx dx_m \quad (D-6)$$

In the first term on the right-hand side of this equation,  $X < T_L$ , an outcome called a lower limit pass error; in the second term,  $X > T_U$ , an outcome called an upper limit pass error. The probabilities of these two outcomes can, if desired, be calculated separately.

### D-8.2 Probability of a Fail Error: Producer's Risk, $R_p$

A fail error occurs when measurement of a conforming workpiece results in a measured value outside of the gaging limits. For such an incorrect decision,  $T_L \leq X \leq T_U$  and  $X_m < G_L$  or  $X_m > G_U$ , and the integral over  $x_m$  in eq. (D-1) thus consists of two parts: one from  $-\infty$  to  $G_L$  and another from  $G_U$  to  $\infty$ .

The producer's risk is therefore calculated as follows:

$$R_p = \int_{-\infty}^{G_L} \int_{T_L}^{T_U} f(x, x_m) dx dx_m + \int_{G_U}^{\infty} \int_{T_L}^{T_U} f(x, x_m) dx dx_m \quad (D-7)$$

In the first term on the right-hand side of eq. (D-7),  $X_m < G_L$ , an outcome called a lower limit fail error; in the second term,  $X_m > G_U$ , an outcome called an upper limit fail error. The probabilities of these two outcomes can, if desired, be calculated separately.

### D-8.3 Probabilities of Accepting a Conforming Workpiece and Rejecting a Nonconforming Workpiece

For completeness, the probabilities of the two remaining outcomes of a binary measurement decision are discussed here.

Accepting a conforming workpiece means that  $T_L \leq X \leq T_U$  and  $G_L \leq X_m \leq G_U$ . From eq. (D-1), the probability of this outcome is

$$\text{Pr (accept conforming workpiece)} = \int_{G_L}^{G_U} \int_{T_L}^{T_U} f(x, x_m) dx dx_m \quad (D-8)$$



Rejecting a nonconforming workpiece means that  $X < T_L$  or  $X > T_U$ , and  $X_m < G_L$  or  $X_m > G_U$ . The probability of this outcome can be calculated, using eq. (D-1), as a sum of four integrals with appropriate limits. It is easier, however, to note that one of the four possible outcomes is certain to occur, so that

$$\Pr(\text{reject nonconforming workpiece}) = 1 - \Pr(\text{accept conforming workpiece}) = R_c + R_p \quad (\text{D-9})$$

with the three probabilities on the right-hand side given by eqs. (D-8), (D-6), and (D-7).

#### D-8.4 Example

The risk calculations discussed above are illustrated by the following case:

(a) The process distribution is a Gaussian PDF [see eq. (D-3)], with mean value,  $x_0 = 1\,500$  mm and standard deviation,  $u_p = 0.121$  mm.

(b) The measuring system distribution is a Gaussian PDF [see eq. (D-4)], with standard uncertainty,  $u_m = 0.04$  mm.

(c) The tolerance and gaging limits are

$$\begin{aligned} G_L &= 1\,499.82 \text{ mm} \\ G_U &= 1\,500.18 \text{ mm} \\ T_L &= 1\,499.8 \text{ mm} \\ T_U &= 1\,500.2 \text{ mm} \end{aligned}$$

The decision rule in this example (see ASME B89.7.3.1 [19]) is stringent acceptance, relaxed rejection, with symmetric  $100 \times (T_U - G_U)/(2u_m) = 25\%$  guard bands (i.e.,  $g_L = g_U = 0.25 \times U_{k=2}$ ).

Inserting the example data into eq. (D-5) yields the explicit form of the joint PDF for  $X$  and  $X_m$

$$f(x, x_m) = 32.9 \times \exp - \left[ \frac{(x - 1\,500)^2}{0.029 \text{ mm}^2} + \frac{(x_m - x)^2}{0.0032 \text{ mm}^2} \right] \quad (\text{D-10})$$

The probabilities of the four possible outcomes are then calculated by integrating the joint density (D-10) over the appropriate limits as discussed above.

##### D-8.4.1 Outcomes

(a) *Consumer's Risk*

$$\begin{aligned} R_c &= \int_{1499.82}^{1500.18} \int_{-\infty}^{1499.8} f(x, x_m) dx dx_m + \int_{1499.82}^{1500.18} \int_{1500.2}^{\infty} f(x, x_m) dx dx_m \\ &= 0.005 + 0.005 \\ &= 1\% \end{aligned} \quad (\text{D-11a})$$

(b) *Producer's Risk*

$$\begin{aligned} R_p &= \int_{-\infty}^{1499.82} \int_{1499.8}^{1500.2} f(x, x_m) dx dx_m + \int_{1500.18}^{\infty} \int_{1499.8}^{1500.2} f(x, x_m) dx dx_m \\ &= 0.035 + 0.035 \\ &= 7\% \end{aligned} \quad (\text{D-11b})$$

(c) *Probability of Accepting a Conforming Workpiece*

$$\begin{aligned} \Pr(\text{accept conforming workpiece}) &= \int_{1499.82}^{1500.18} \int_{1499.8}^{1500.2} f(x, x_m) dx dx_m \\ &= 0.832 \\ &= 83.2\% \end{aligned} \quad (\text{D-11c})$$



(d) *Probability of Rejecting a Nonconforming Workpiece*

$$\begin{aligned}\Pr(\text{reject nonconforming workpiece}) &= 1 - R_c - R_p - \Pr(\text{accept conforming workpiece}) \\ &= 0.088 \\ &= 8.8\%\end{aligned}\quad (\text{D-11d})$$

Note that because of the symmetric guard bands, the consumer's and producer's risks are each composed of equal contributions due to upper and lower pass and fail errors.

## D-9 ONE-SIDED MEASUREMENTS

Figure D-1-2 illustrates a typical one-sided measurement decision problem. In the case where a measured property of interest is strictly positive (such as flatness), the lower limit of the tolerance interval is zero and the upper limit is specified by the tolerance,  $T$ . The probabilities of pass errors and fail errors are controlled by choice of a gaging limit,  $G$ . Calculation of these probabilities is similar to the approach of section D-8, except for the nature of the process distribution.

### D-9.1 Process Distribution

The form of the process distribution,  $p_0(x)$ , is assigned based on knowledge of the manufacturing process. Such knowledge is often summarized by the mean,  $\bar{x}$ , and standard deviation,  $s$ , of a set of measurements made on a sample of workpieces. These summary parameters can then be used in formulation of a suitable form for  $p_0(x)$ . Some reasonable possibilities include

(a) In the case where typical values of the measurand,  $X$ , are close to zero, the process can be modeled by a half-Gaussian (half-normal) distribution, which is a Gaussian PDF with maximum value at zero, for positive values of  $x$

$$p_0(x) = \frac{1}{\sigma} \sqrt{\frac{2}{\pi}} \exp(-x^2 / 2\sigma^2), \quad x \geq 0 \quad (\text{D-12})$$

In this equation,  $\sigma$  is not a standard deviation but rather a parameter that sets the scale of the distribution. The mean,  $x_0$ , and standard deviation,  $u_p$ , of the half-Gaussian PDF are given by

$$x_0 = \sigma \sqrt{\frac{2}{\pi}} \approx 0.8\sigma \text{ and } u_p = \sigma \sqrt{1 - \frac{2}{\pi}} \approx 0.6\sigma$$

It might be reasonable to use the summary parameters  $\bar{x}$  and  $s$  to estimate  $x_0$  and  $u_p$ , and thus to calculate a compromise value of  $\sigma$  to characterize the process.

(b) Another process distribution that might be chosen when values of  $X$  close to zero are likely is the exponential distribution

$$p_0(x) = be^{-bx}, \quad x \geq 0 \quad (\text{D-13})$$

For this distribution, the mean,  $x_0$ , and standard deviation,  $u_p$ , are given by

$$x_0 = 1/b \text{ and } u_p = 1/b^2$$

(c) If the production process is such that very small values of a property of interest are rarely observed, ASME B89.7.4.1 suggests use, if appropriate, of a gamma probability density,  $g(x; a, b)$ , defined by

$$p_0(x) = g(x; a, b) = \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx}, \quad x \geq 0 \quad (\text{D-14})$$

Here,  $a$  and  $b$  are two positive parameters and  $\Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx$  is the gamma function. The mean,  $x_0$ , and standard deviation,  $u_p$ , of the gamma PDF are given by

$$x_0 = \frac{a}{b} \text{ and } u_p = \frac{\sqrt{a}}{b}$$



(d) Note that for  $a = 1$ ,  $\Gamma(1) = 1$  and the gamma PDF reduces to the exponential distribution with parameter  $b$  given by eq. (D-13).

### D-9.2 Measuring System Distribution

A Gaussian distribution is usually a reasonable choice to characterize knowledge of the measuring system; see eq. (D-4). For a true value,  $X$ , close to zero, such a distribution would assign a non-negligible probability to possible measured values,  $x_m$ , that are less than zero. Such an outcome is not possible for measurements of intrinsically positive workpiece properties such as straightness or roundness.

The probability of a negative measured value can be reduced to zero by truncating the Gaussian distribution to exclude values less than zero. Such a truncated distribution is written as

$$p(x_m | x) = K^{-1} \exp \left[ -\frac{1}{2} \left( \frac{x_m - x}{u_m} \right)^2 \right], \quad 0 \leq x_m < \infty \quad (\text{D-15})$$

where  $K$  is a constant that ensures that the area under the distribution is equal to one. Explicitly

$$\begin{aligned} K &= \int_0^{\infty} \exp \left[ -\frac{1}{2} \left( \frac{x_m - x}{u_m} \right)^2 \right] dx_m \\ &= u_m \sqrt{2\pi} [1 - \Phi(-x / u_m)] \end{aligned} \quad (\text{D-16})$$

In the last equation,  $\Phi$  is the standard normal cumulative distribution function defined by

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^z \exp(-t^2 / 2) dt \quad (\text{D-17})$$

Note that the normalization constant,  $K$ , has a different value for each true value,  $x$ .

Risk calculations using a truncated Gaussian measuring system distribution may be carried out using appropriate numerical analysis software. Formulae for the various risk probabilities are given below.

### D-9.3 Probability of a Pass Error

For a one-sided measurement, a pass error occurs for  $0 \leq X_m \leq G$  and  $X > T$ . Given the joint PDF  $f(x, x_m) = p_0(x)p(x_m | x)$ , with  $p_0(x)$  assigned based on knowledge of the process, the probability of a pass error (consumer's risk) is given by

$$R_c = \int_0^G \int_T^{\infty} f(x, x_m) dx dx_m \quad (\text{D-18})$$

### D-9.4 Probability of a Fail Error

A fail error occurs when  $X_m < 0$  or  $X_m > G$  and  $0 \leq X \leq T$ . The probability of such an outcome (producer's risk) is given by

$$R_p = \int_{-\infty}^0 \int_0^T f(x, x_m) dx dx_m + \int_0^T \int_G^{\infty} f(x, x_m) dx dx_m \quad (\text{D-19})$$

### D-9.5 Probability of Accepting a Conforming Workpiece

Accepting a conforming workpiece results when  $0 \leq X_m \leq G$  and  $0 \leq X \leq T$ . The probability of this outcome is

$$\Pr(\text{accept conforming workpiece}) = \int_0^T \int_0^G f(x, x_m) dx dx_m \quad (\text{D-20})$$



### D-9.6 Probability of Rejecting a Nonconforming Workpiece

Rejection of a nonconforming workpiece results when  $X_m < 0$  or  $X_m > G$  and  $X > T$ . Since one of the four possible outcomes is certain to occur, the probability of rejecting a nonconforming workpiece is

$$\Pr(\text{reject nonconforming workpiece}) = 1 - R_c - R_p - \Pr(\text{accept conforming workpiece}) \quad (\text{D-21})$$

### D-10 COMBINING PROBABILITIES

Suppose that several features of a workpiece are measured. There may be a need to know the probabilities of a pass error (consumer's risk) and a fail error (producer's risk) for the workpiece as a whole, without regard to which features might be conforming or nonconforming. Such risk calculations, as illustrated below, make use of well known rules for combining probabilities that are developed in books on probability and statistics; see, for example, reference [22].

In the simplest case, consider a workpiece with two dimensions of interest, such as a rectangular block whose length and width are specified by nominal values, together with associated tolerances. The workpiece will be nonconforming if either of the two dimensions (or both) are outside their respective tolerance intervals.

Denote by  $e_1$  and  $e_2$  events corresponding to pass errors of dimensions one and two, respectively. The workpiece as a whole will be mistakenly accepted (i.e., a workpiece pass error) if either  $e_1$  or  $e_2$  (or both) occurs. The probability of this outcome is

$$\begin{aligned} \Pr(\text{workpiece pass error}) &= \Pr(e_1 \text{ or } e_2) \\ &= \Pr(e_1) + \Pr(e_2) - \Pr(e_1 \text{ and } e_2) \end{aligned} \quad (\text{D-22})$$

The last term on the right is the probability that pass errors occur on both dimensions during workpiece inspection, which can be written as

$$\Pr(e_1 \text{ and } e_2) = \Pr(e_1) \Pr(e_2 | e_1) \quad (\text{D-23})$$

The quantity  $\Pr(e_2 | e_1)$  is the conditional probability of a pass error on dimension two, given that a pass error has occurred on dimension one. The magnitude of this quantity depends on the degree of correlation between the two pass errors. In most cases it can be expected that the two errors are uncorrelated, for the following reason.

Consider the particular case where errors  $e_1$  and  $e_2$  are upper limit pass errors on dimensions one and two. Such an error occurs when a true dimension exceeds its upper tolerance limit, while measurement of the dimension yields a measured value within the acceptance interval.

In order for errors  $e_1$  and  $e_2$  to be logically correlated, two things must happen: knowledge that dimension one is too large must imply that dimension two is also too large, and knowledge that a measurement error on dimension one results in an acceptable measured value must imply a corresponding measurement error on dimension two.

The conditions above imply specific knowledge of production bias in the manufacturing system, together with specific knowledge of measurement bias in the measuring system. For typical production/measurement processes, it is unlikely that such specific knowledge will exist, particularly when noise and nonrepeatability are significant contributors to these processes.

The probability of a pass error for the workpiece as a whole will thus be calculated only for the case of uncorrelated pass errors  $e_1$  and  $e_2$ .

If pass errors  $e_1$  and  $e_2$  are uncorrelated, then occurrence of  $e_1$  is irrelevant in considering the probability of  $e_2$ , so that  $\Pr(e_2 | e_1) = \Pr(e_2)$ .

Then, from eqs. (D-22) and (D-23)

$$\Pr(\text{workpiece pass error}) = \Pr(e_1) + \Pr(e_2) - \Pr(e_1)\Pr(e_2) \quad (\text{D-24})$$

The probability of a pass error is the consumer's risk, so that in the case of uncorrelated errors, eq. (D-24) becomes

$$R_c(\text{uncorrelated}) = R_{c_1} + R_{c_2} - R_{c_1}R_{c_2} \quad (\text{D-25})$$

where  $R_{c_1}$  and  $R_{c_2}$  are the consumer's risks for dimensions one and two, respectively.



In the common case where the individual consumer's risks,  $R_{c_1}$  and  $R_{c_2}$ , are on the order of a few percent, the total consumer's risk can be taken, with negligible error, to be the sum of the individual risks

$$R_c(\text{uncorrelated}) \approx R_{c_1} + R_{c_2} \quad (\text{D-26})$$

Given  $R_{c_1}$  and  $R_{c_2}$  each equal to 5%, for example, from eq. (D-25), the total consumer's risk is

$$R_c = 0.05 + 0.05 - (0.05)^2 = 0.0975 \approx 10\% \quad (\text{D-27})$$

This approach can be generalized to the case of a workpiece with several dimensions of interest, when the individual pass errors and fail errors are uncorrelated. By analogy with eq. (D-25), when none of the individual risks is greater than a few percent, the total consumer's risk will be approximately equal to the sum of the individual consumer's risks, with a similar result for the total producer's risk. More generally, since probabilities are non-negative numbers, the sum of the individual consumer's risks is always an upper bound on the total consumer's risk regardless of the magnitude and correlation of the individual pass errors.



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